Sensitivity and decomposition of multistate healthy life expectancy

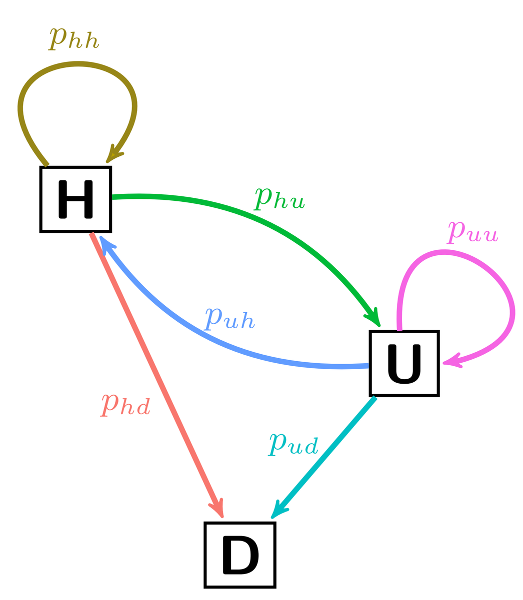
Problem

Did you know that the leverage of a given transition probability on healthy life expectancy (HLE) depends on which other transitions you use to calcualte it?

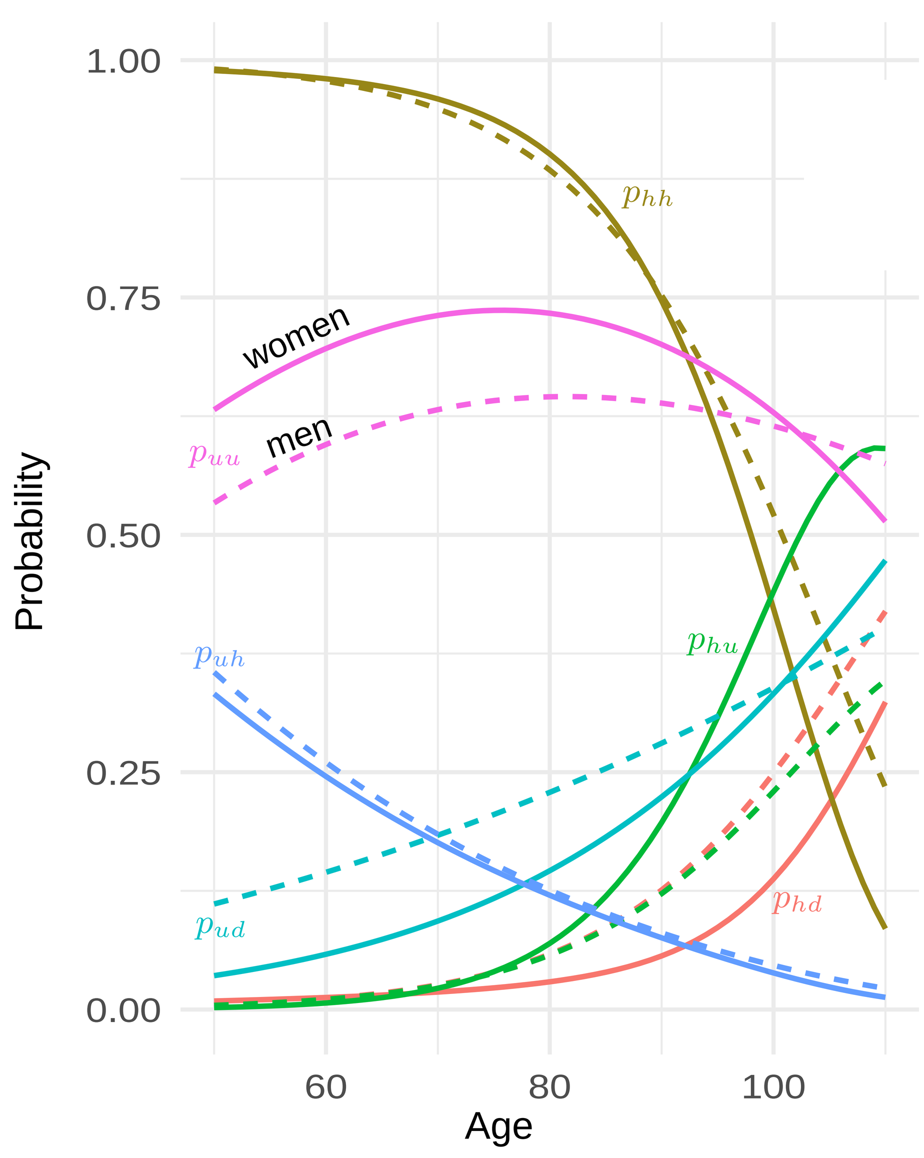
Allow me to make a case for framing decomposition results in terms of **attrition** parameters only.

Setup

Start with this simple state space: **H**ealthy, **U**nhealthy, **D**ead..



Let's use these transitions from Lievre et. al. (2003).



Using these transitions and some simple assumptions, we get the following expectancies at age 50:

	HLE	ULE	LE
men	25.69	1.67	27.37
women	27.15	2.84	29.99

It's only natural to ask why women live 1.46 years longer in good health, 1.17 years longer in poor health, or 2.62 years longer in total. That is, we'd like to be able to **decompose** the difference. The problem is less straightforward than it ought to be, given that these values are **fully determined** by the above transitions.

Decomposition results depend on *which* parameters are used to calcualte the expectancy. Let's consider, three **cases**, or ways to *functionalize* the problem:

The cases

1 $f1(p_{hh}, p_{hu}, p_{uh}, p_{uu})$

$$l_h(a+1) = l_h(a)p_{hh}(a) + l_u(a)p_{uh}(a)$$
$$l_u(a+1) = l_u(a)p_{uu}(a) + l_h(a)p_{hu}(a)$$

2 $f2(p_{hd}, p_{hu}, p_{uh}, p_{ud})$

$$l_h(a+1) = l_h(a) - l_h(a)(p_{hu}(a) + p_{hd}(a)) + l_u(a)p_{uh}(a)$$
$$l_u(a+1) = l_u(a) - l_u(a)(p_{uh}(a) + p_{ud}(a)) + l_h(a)p_{hu}(a)$$

3 $f3(p_{hh}, p_{hd}, p_{ud}, p_{uu})$

$$l_h(a+1) = l_h(a)p_{hh}(a) + l_u(a) - l_u(a)(p_{ud}(a) + p_{uu}(a))$$
$$l_u(a+1) = l_u(a)p_{uu}(a) + l_h(a) - l_h(a)(p_{hd}(a) + p_{hh}(a))$$

From each of these, we can generate state-specific or total life expectancy

$$HLE = \sum_a l_h(a)$$
$$ULE = \sum_a l_u(a)$$
$$LE = HLE + ULE$$

You get **identical** expectancies no matter the case

Discrepancies

The **sensitivity** of the expectancy **depends on the *case*** which also means the decomposition results do. In this poster we just show the sensitivities. In general, you can decompose by taking the sum product of the parameter difference and the parameter sensitivity. You can either calculate the sensitivities directly or use a numerical gradient estimator (for example in R: `numDeriv::grad()`). We give analytical solutions for the sensitivities to be able to demonstrate certain symmetries.

Skip the math (see paper) and just look at the sensitivities! Each sensitivity line is labelled with the transition parameter it refers to.

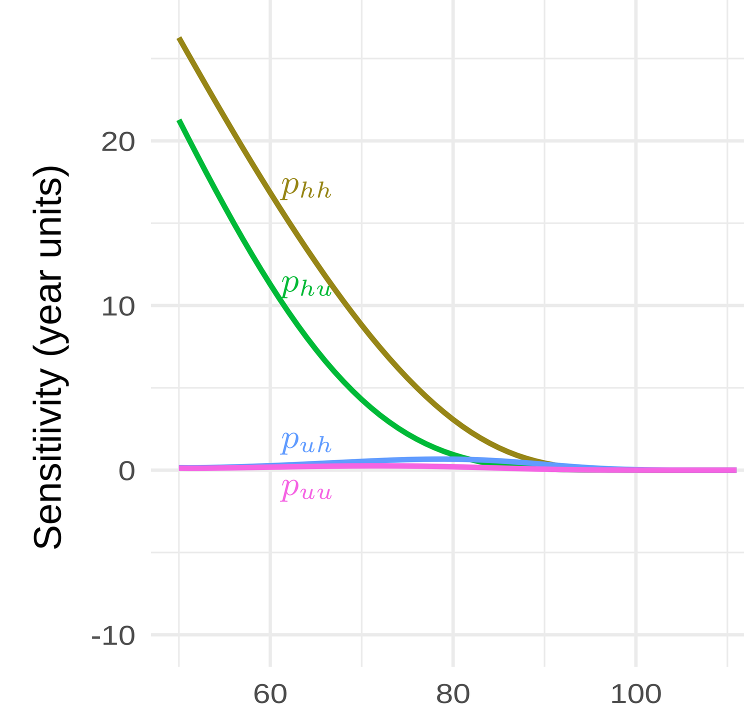
Case 1

If you take the set of transitions required by matrix algebra calculations (case 1) to be the full story, then you'd generate this result and say "job done". In effect, the excluded parameters (mortality here) are treated as a residual.

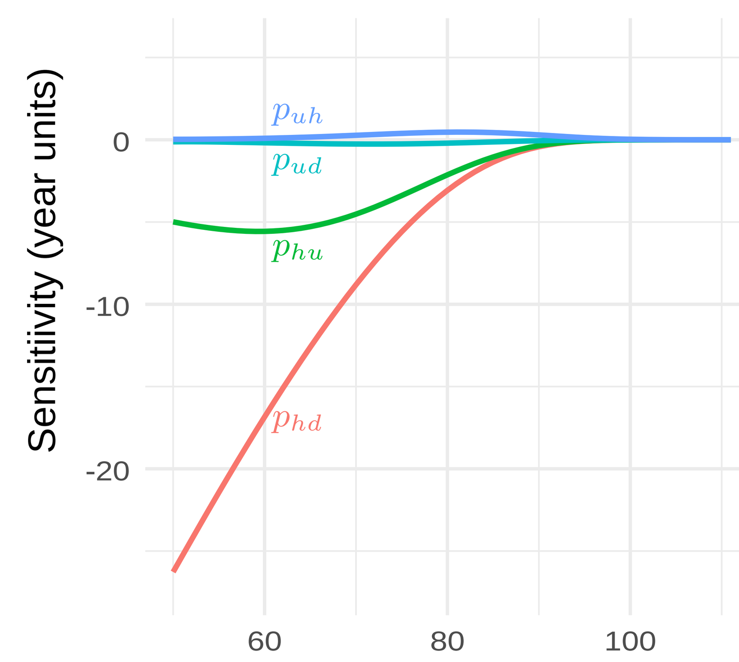
At face value, you'd conclude that the most effective way to increase HLE is to.. 🥁 keep people healthy? That doesn't seem very specific.

It's not as straightforward as relabelling the p_{hh} sensitivity as p_{hd} . Look at p_{hd} from case 2 and you'll see that these are the **same magnitude** but **opposite sign**. Then look closely at p_{hd} from case 3 and notice it's a different magnitude from case 2. Look at p_{hu} in cases 1 and 2 and you'll see that they have different signs, magnitudes, and age patterns.

On it's own, case 1 sensitivities are problematic in their interpretation.



Case 2



Case 2 parameters are all forms of attrition. As such, they are competing risks. We're primed to think of competing risks as problematic and not independent. But this case is really not very different from cause-of-death decompositions. Attrition transitions seem more independent to us than, say, dying in the health state versus staying in the health state (case 1). And at least in continuous time, competing risks should vanish.

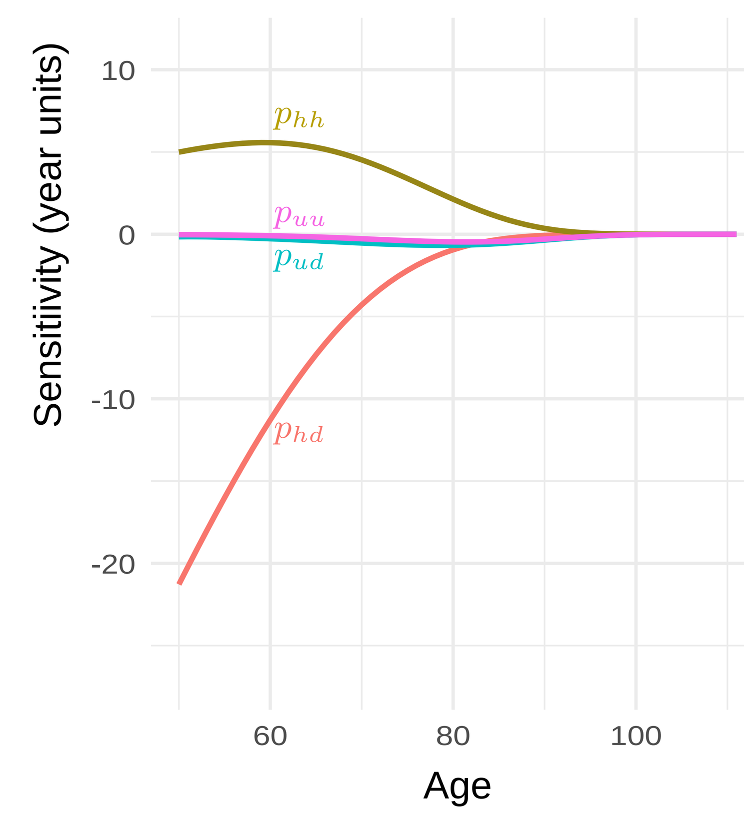
Also observe: the interpretation of case 2 sensitivities (and decompositions) conforms with (but also adds detail to) Sullivan decompositions: everything is either mortality or health. This plays well with the existing literature.

I lean toward **this framing** (or variations on it*) **as the best for health models**.

(*) At REVES 2021 I suggested a different conditional framing, which gives very similar results to case 2. See this video, minute 13:30

Case 3

Case 3 parameters are silly: **A health model with no health transitions?!** Literally no one has ever calculated HLE this way, but it is just as valid as the others. You can **transfer your indignation** at the lack of health in case 3 to the lack of mortality in case 1! Also notice that p_{hh} here has a different shape and magnitude from case 1!



Symmetries & equalities

Do you now also want to present decomposition results in terms of attrition parameters? What if you're all set up to calculate HLE using matrix algebra and don't want to re-do your workflow? Luckily, there are relationships that you can exploit to translate a sensitivity (or decomposition) from one case to another! Let's look specifically at translating case 1 sensitivities to case 2 (more in the paper):

$$s^2_{hu}(a) = s^1_{hu}(a) - s^1_{hh}(a)$$
$$s^2_{uh}(a) = s^1_{uh}(a) - s^1_{uu}(a)$$
$$s^2_{hd}(a) = -s^1_{hh}(a)$$
$$s^2_{ud}(a) = -s^1_{uu}(a)$$

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Paper

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