Sensitivity and decomposition of multistate healthy life expectancy

Problem
Did you know that the leverage of a given transition probability on healthy life expectancy (HLE) depends on which other transitions you use to calculate it?

Allow me to make a case for framing decomposition results in terms of attrition parameters only.

Setup
Start with this simple state space: Healthy, Unhealthy, Dead...

Let's use these transitions from Lieve et al. (2003).

Using these transitions and some simple assumptions, we get the following expectancies at age 50:

<table>
<thead>
<tr>
<th></th>
<th>HLE</th>
<th>ULE</th>
<th>LE</th>
</tr>
</thead>
<tbody>
<tr>
<td>men</td>
<td>25.69</td>
<td>1.67</td>
<td>27.37</td>
</tr>
<tr>
<td>women</td>
<td>27.15</td>
<td>2.84</td>
<td>29.99</td>
</tr>
</tbody>
</table>

It's only natural to ask why women live 1.46 years longer in good health, 1.17 years longer in poor health, or 2.62 years longer in total. That is, we'd like to decompose the difference. The problem is less straightforward than it ought to be, given that these values are fully determined by the above transitions.

Decomposition results depend on which parameters are used to calculate the expectancy. Let's consider, three cases, or ways to functionalize the problem:

The cases

1. Case 1

\[ f_1(\phi_1, \phi_2, \phi_3, \phi_4) = \phi_1(a+1) - \phi_1(a) = \phi_1(a) + \phi_4(a) \]

2. Case 2

\[ f_2(\phi_1, \phi_2, \phi_3, \phi_4) = \phi_1(a+1) - \phi_1(a) = \phi_1(a) + \phi_4(a) + \phi_2(a) + \phi_3(a) \]

3. Case 3

\[ f_3(\phi_1, \phi_2, \phi_3, \phi_4) = \phi_1(a+1) - \phi_1(a) = \phi_1(a) + \phi_4(a) + \phi_2(a) + \phi_3(a) \]

From each of these, we can generate state-specific or total life expectancy

\[ HLE = \sum a \phi_1(a) \]
\[ ULE = \sum a \phi_2(a) \]
\[ LE = HLE + ULE \]

You get identical expectancies no matter the case

Discrepancies
The sensitivity of the expectancy depends on the case which also means the decomposition results do. In this paper we just show the sensitivities, in general, you can decompose by taking the sum product of the parameter difference and the parameter sensitivity. You can either calculate the sensitivities directly or use a numerical gradient estimator (for example in R: numDeriv::grad()). We give analytical solutions for the sensitivities to be able to demonstrate certain symmetries.

Skip the math (see paper) and just look at the sensitivities! Each sensitivity line is labelled with the transition parameter it refers to.

Case 1

If you take the set of transitions required by matrix algebra calculations (case 1) to be the full story, then you'd generate this result and say "job done". In effect, the excluded parameters (mortality here) are treated as a residual.

At face value, you'd conclude that the most effective way to increase HLE is to keep people healthy? That doesn't seem very specific.

It's not as straightforward as relabelling the \( \phi_1 \) sensitivity as \( \phi_2 \). Look at \( \phi_3 \) from case 2 and you'll see that these are the same magnitude but opposite sign. Then look closely at \( \phi_4 \) from case 3 and notice it's a different magnitude from case 2. Look at \( \phi_1 \) in cases 1 and 2 and you'll see that they have different signs, magnitudes, and age patterns.

On its own, case 1 sensitivities are problematic in their interpretation.

Case 2

Case 2 parameters are all forms of attrition. As such, they are competing risks. We're primed to think of competing risks as problematic and not independent. But this case is really not very different from cause-of-death decompositions. Attrition transitions seem more independent to us than, say, dying in the health state versus staying in the health state (case 1).

And at least in continuous time, competing risks should seem very specific.

Also observe: the interpretation of case 2 sensitivities (and decompositions) conforms with (but also adds detail to) Sullivan decompositions: everything is either mortality or health. This plays well with the existing literature.

I lean toward this framing (or variations on it*) as the best for health models.

(*) At REVES 2021 I suggested a different conditional framing, which gives very similar results to case 2. See this video, minute 13:30.

Case 3

Case 3 parameters are silly: A health model with no health transitions?! Literally no one has ever calculated HLE this way, but it is just as valid as the others. You can transfer your indignation at the lack of health in case 3 to the lack of mortality in case 1! Also notice that \( \phi_4 \) here has a different shape and magnitude from case 1.

Symmetries & equalities
Do you now also want to present decomposition results in terms of attrition parameters? What if you'd all set up to calculate HLE using matrix algebra and don't want to re-do your workflow? Luckily, there are relationships that you can exploit to translate a sensitivity (or decomposition) from one case to another. Let's look specifically at translating case 1 sensitivities to case 2 (more in the paper):

\[ \frac{\partial HLE}{\partial \phi_1} = \frac{\partial ULE}{\partial \phi_1} - \frac{\partial LE}{\partial \phi_1} \]

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