Hexic (and Other) Representations of the Mortality Curve

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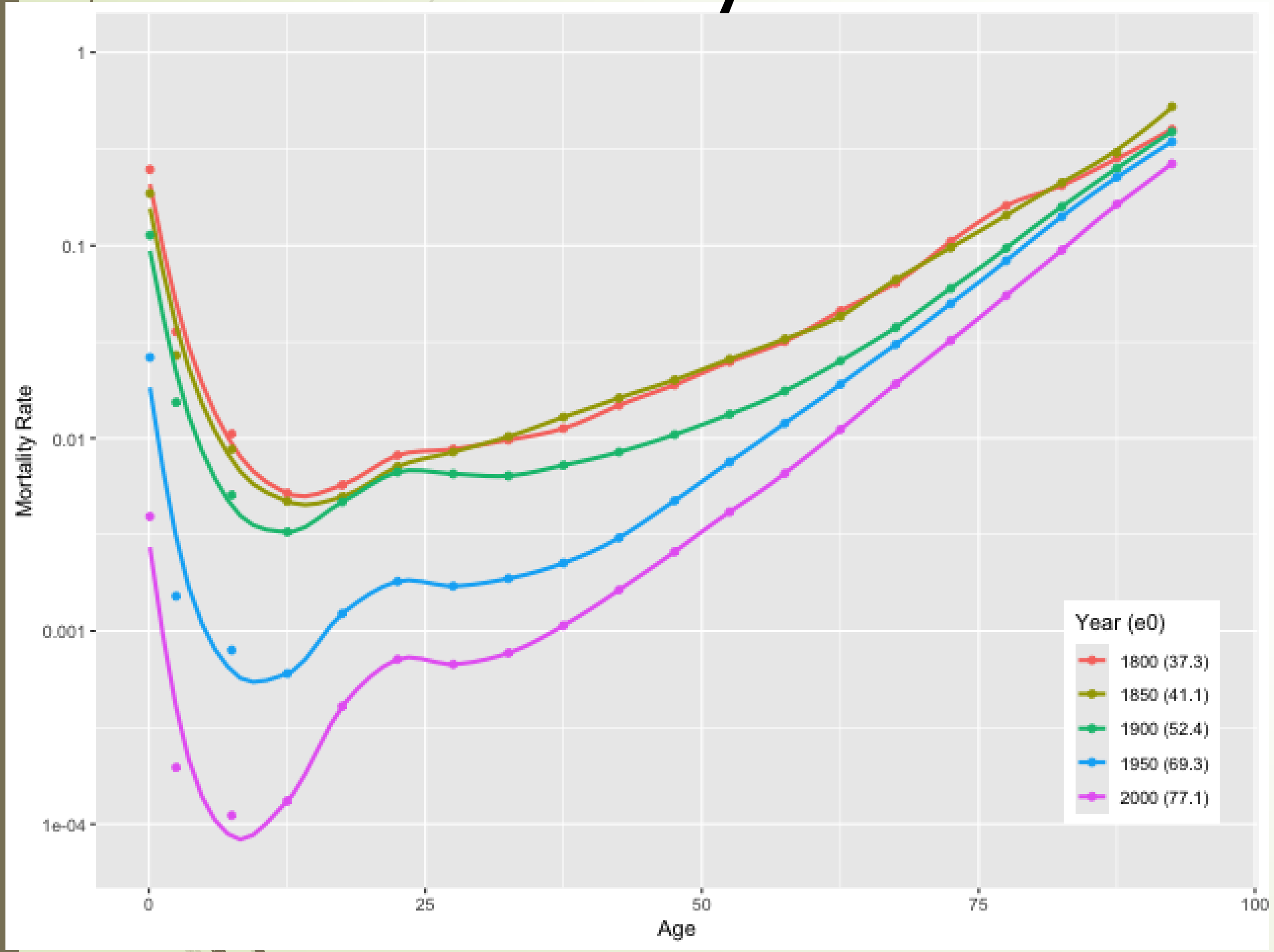
The Question

- As mortality declines, the shape of the mortality curve changes
  - Shift to the right as mortality becomes concentrated in later life
  - Extended depth as minimum value of mortality risk declines
  - Extended domain over which mortality level is minimal

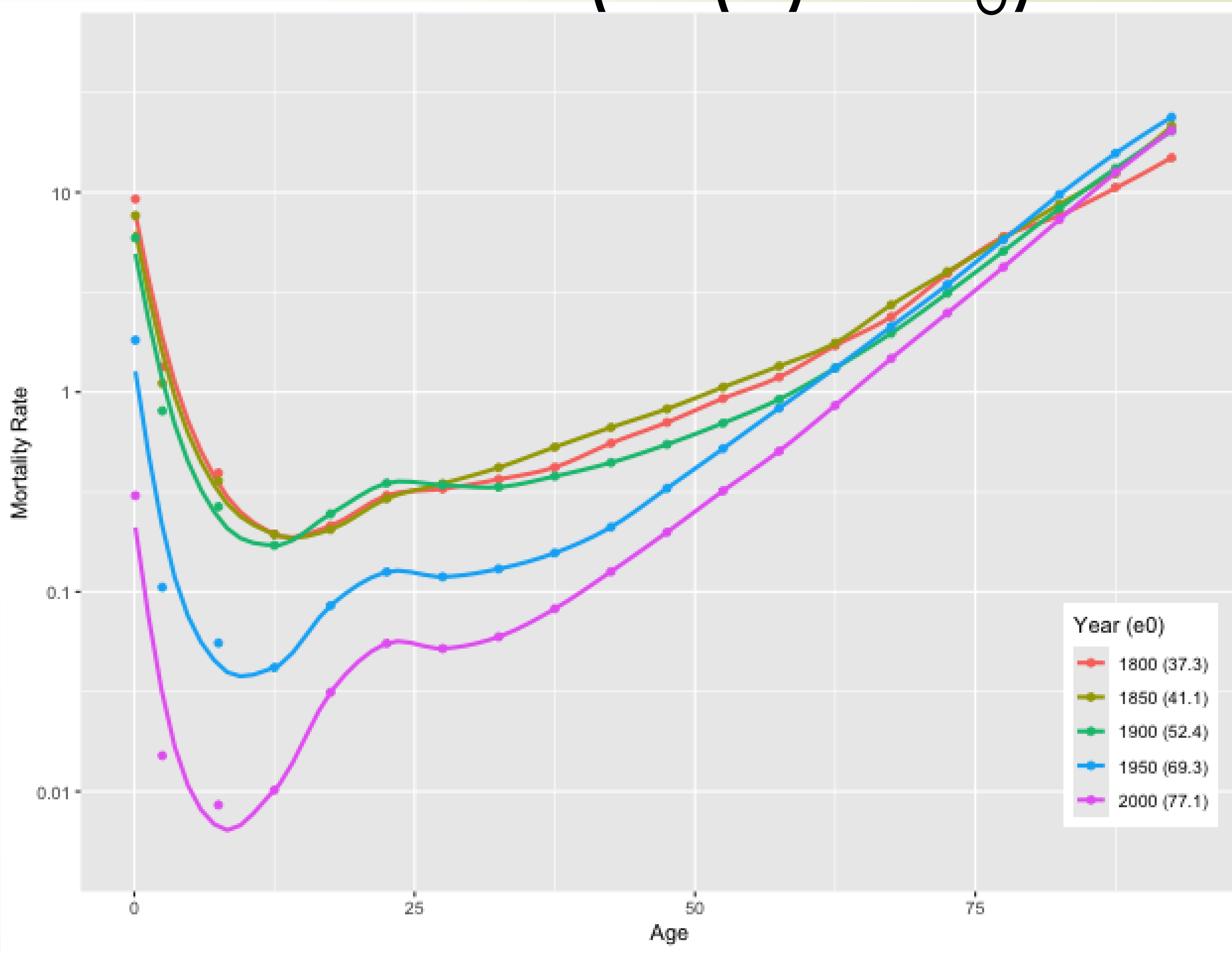
- Not all mortality curves follow the same trajectory
- **Question**: how to disentangle **necessary** changes in shape as mortality declines from **population specific variation**?
- **Corollary:** how to define level of mortality in a manner independent of the contingencies of shape?

Swedish Male Mortality, 1800 to 2000

Mortality Curves



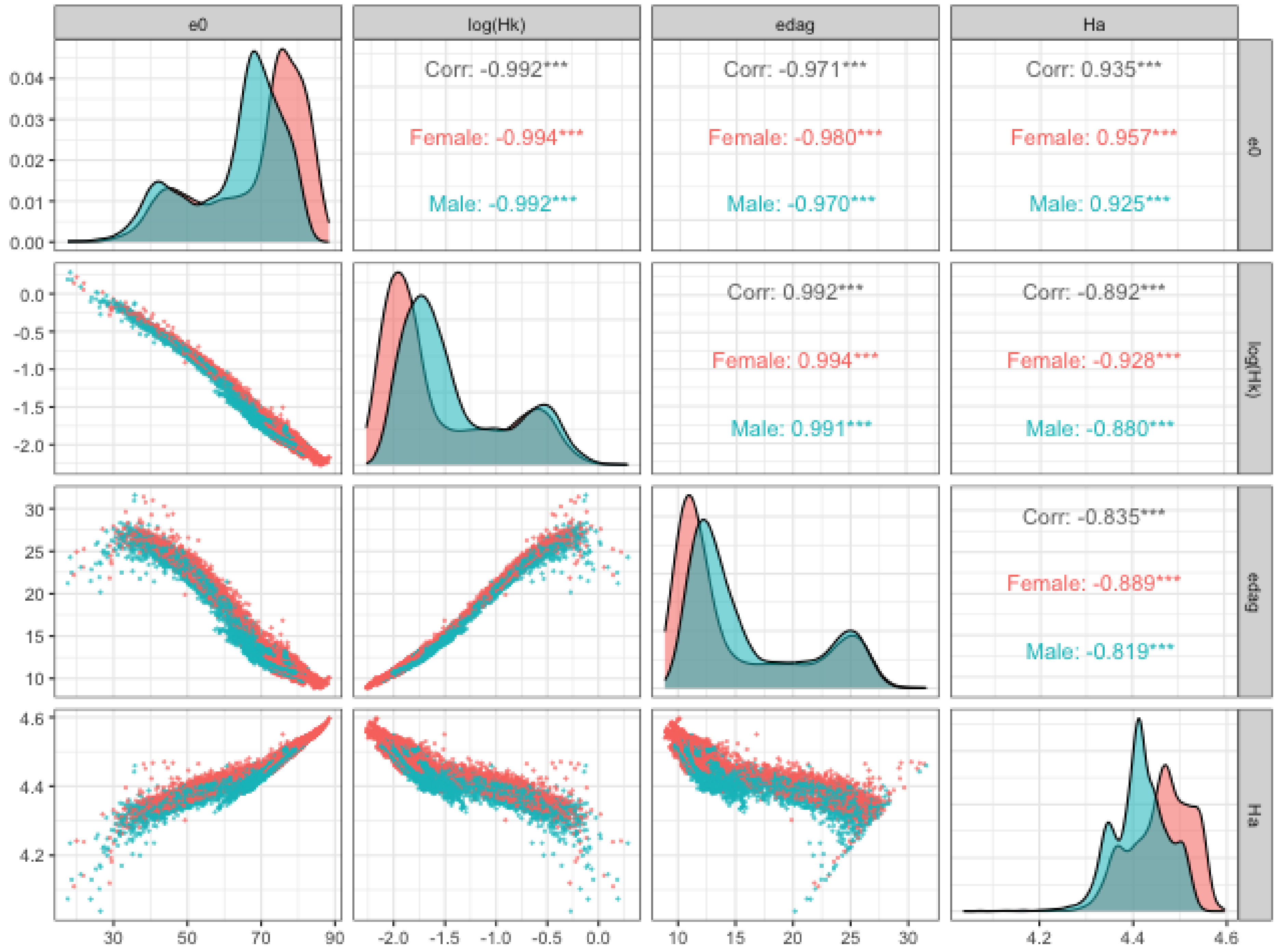
Normalised $(m(x) * e_0)$



Solution 1: Omnibus measures of distribution

- Keyfitz-Golini Rectangularity
  - $H_k = -\frac{1}{e_0}\int_0^{\omega} l(x)\log\big(l(x)\big)\,dx$
- Disparity
  - $e^{\dagger} = -\int_0^{\omega} l(x)\log\big(l(x)\big)\,dx = H_k \cdot e_0$
- Entropy (of age distribution)
  - $H_a = -\int_0^{\omega} \frac{l(x)}{e_0}\log\left(\frac{l(x)}{e_0}\right)dx = H_k + \log(e_0)$

Data: HMD, complete set, 8570 abridged life tables



e0
log(Hk)
edag
Ha
Corr: -0.992***
Female: -0.994***
Male: -0.992***
Corr: -0.971***
Female: -0.980***
Male: -0.970***
Corr: 0.935***
Female: 0.957***
Male: 0.925***
Corr: 0.992***
Female: 0.994***
Male: 0.991***
Corr: -0.892***
Female: -0.928***
Male: -0.880***
Corr: -0.835***
Female: -0.889***
Male: -0.819***

Solution 2: Hexic Curve

The underlying shape of the mortality is a hexic curve:

$$Y = X^6 - \beta X^4 + \phi X^2 + \tau X \quad (1)$$

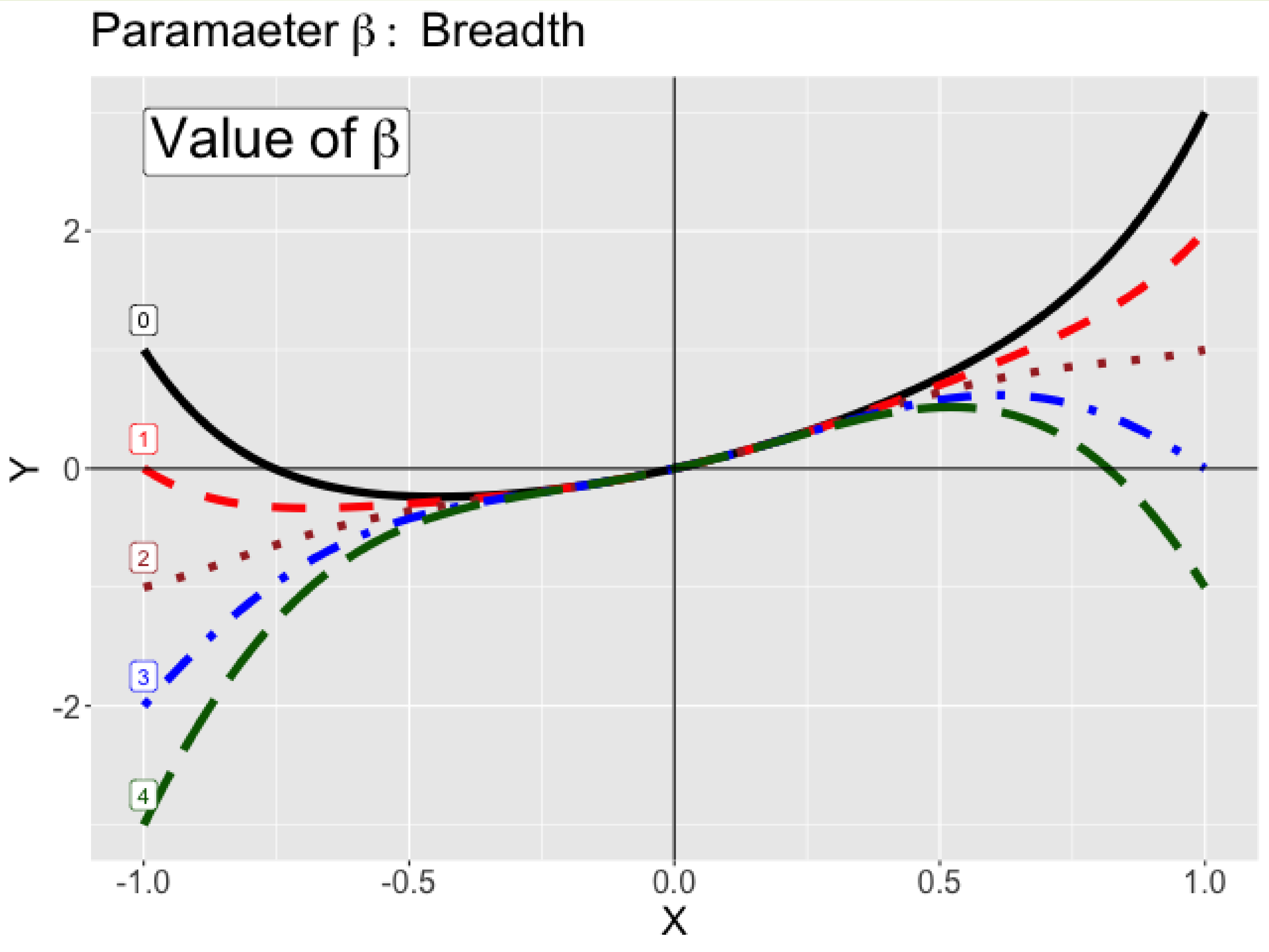
Structural parameters β (Breadth); ϕ (Flatness); τ (Tilt).

Scaled and located on the log(m_x) and age (x) axes by:

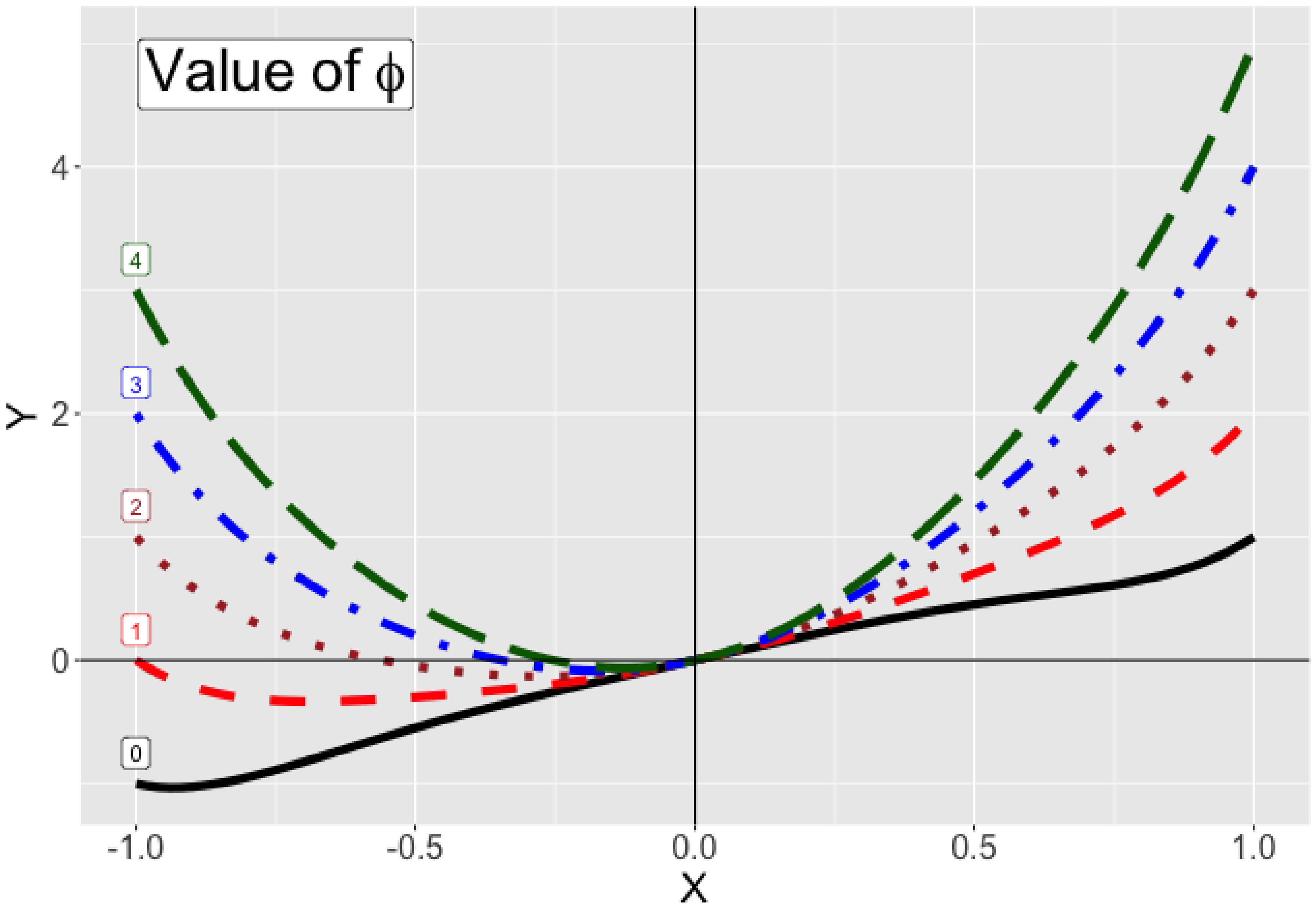
$$\log(m_x) = [\sigma(\xi - x)]^6 - \beta[\sigma(\xi - x)]^4 + \phi[\sigma(\xi - x)]^2 + \tau[\sigma(\xi - x)] - \lambda \quad (2)$$

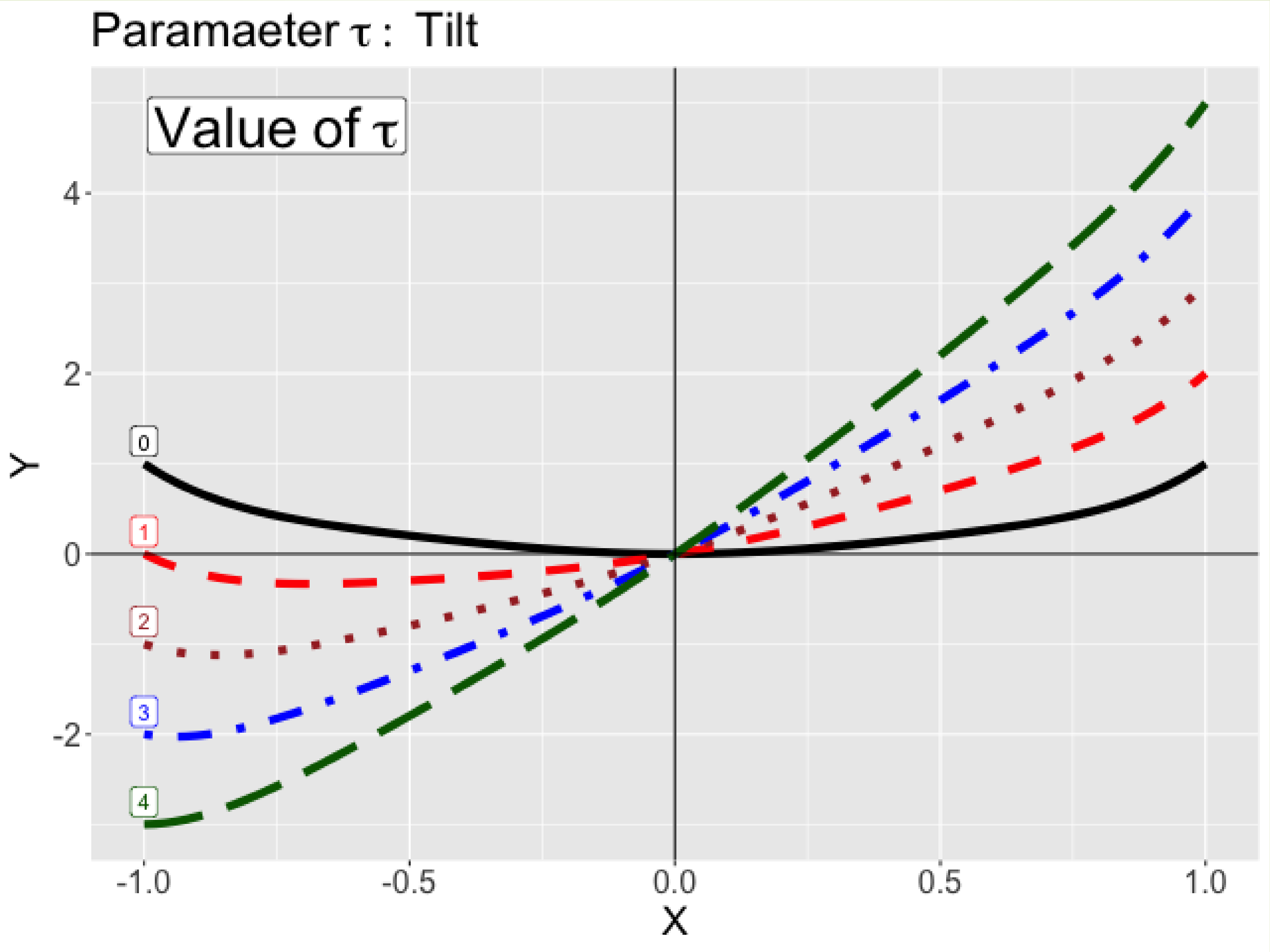
Objectives: To look at

1. How the parameters affect the shape of the curve
2. How they change as mortality declines

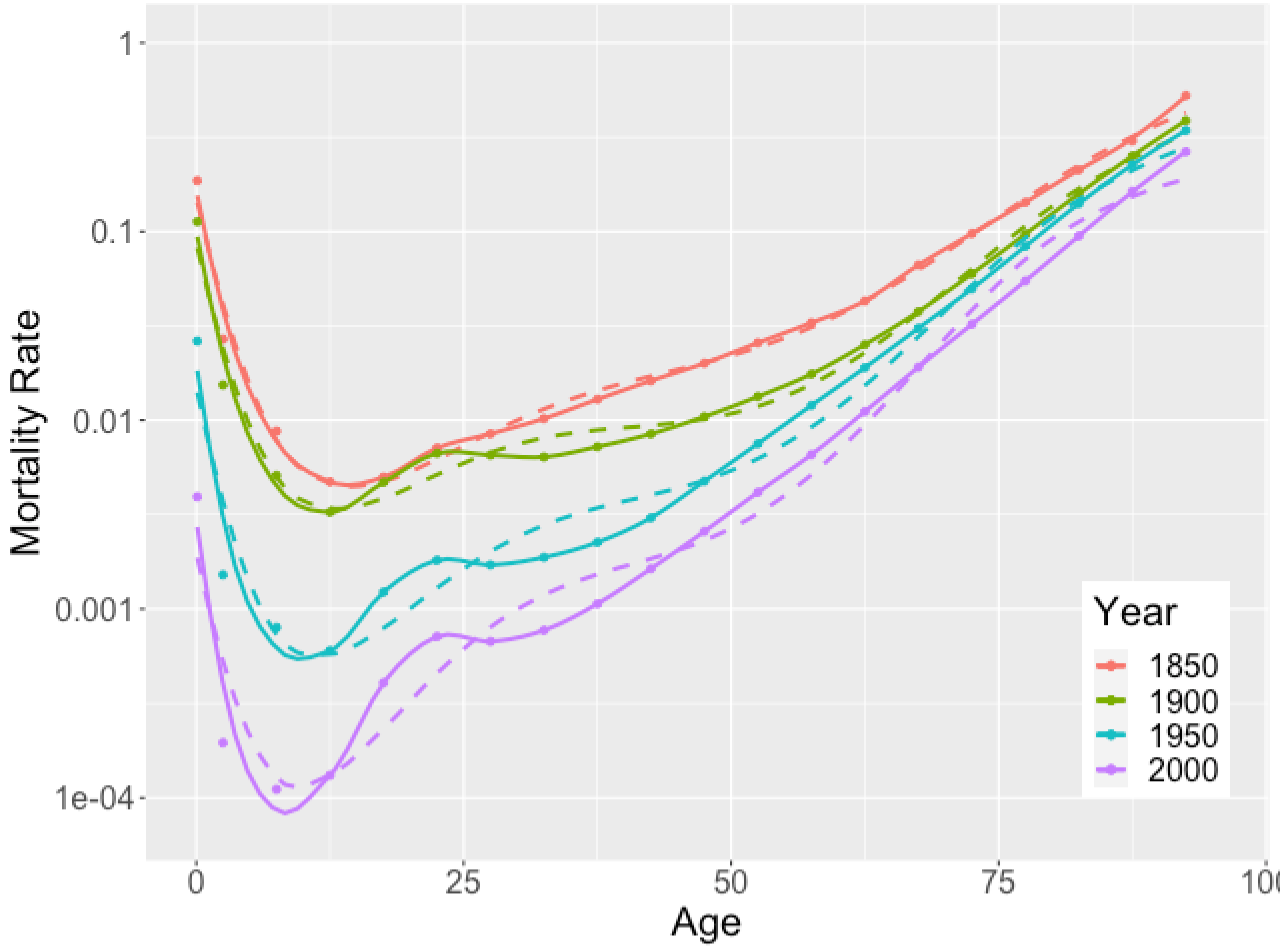

Paramaeter β : Breadth
Value of β
0
1
2
3
4
2
0
-2
Y
-1.0
-0.5
0.0
0.5
1.0
X

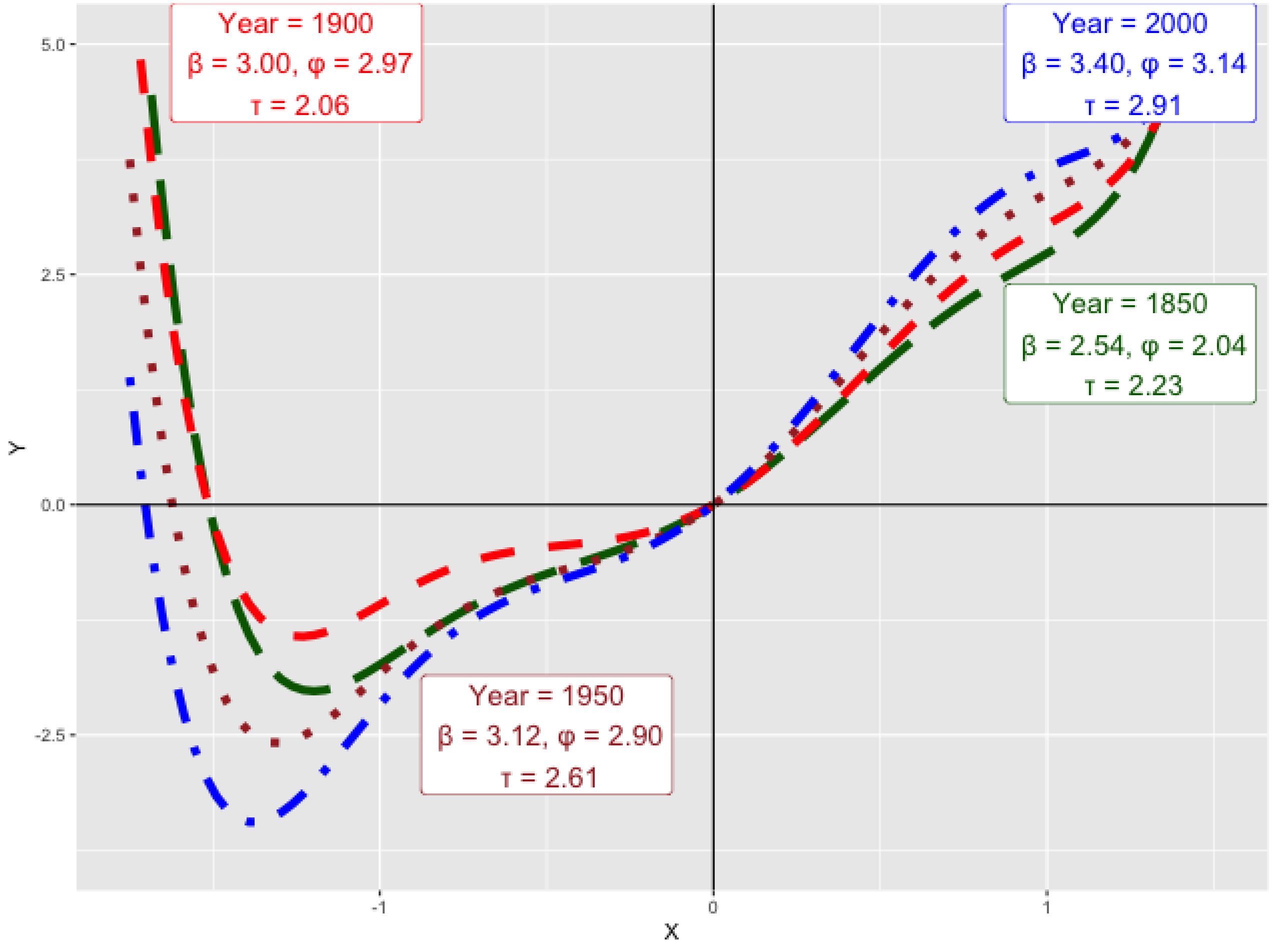
Parameter ϕ : Flatness



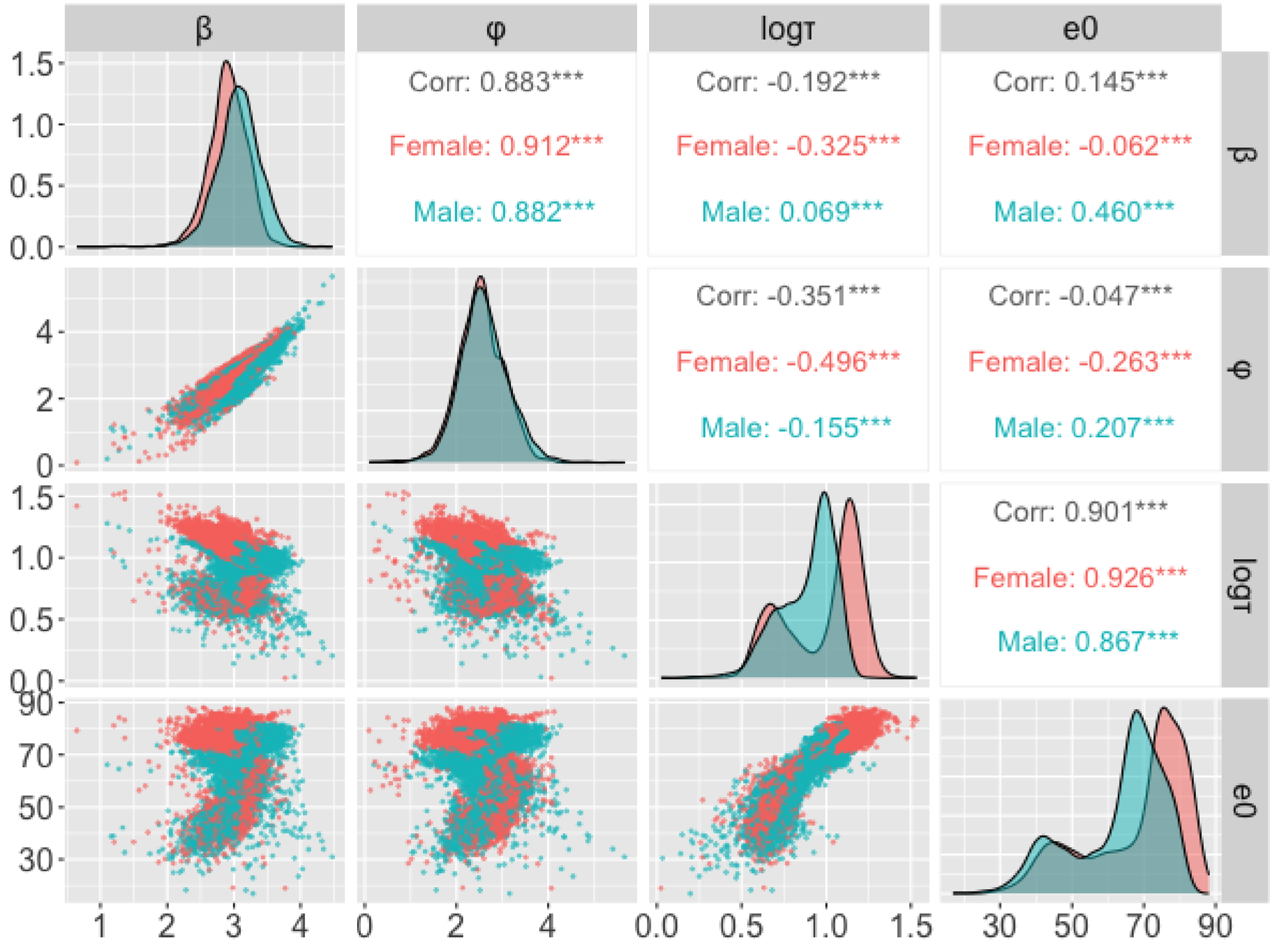

Paramaeter τ : Tilt
Value of τ
0
1
2
3
4
4
2
0
-2
Y
-1.0
-0.5
0.0
0.5
1.0
X

Fit and Structure of Swedish Mortality Curves





Solid lines = data; dashed lines = fitted


β
φ
logτ
e0
Corr: 0.883***
Female: 0.912***
Male: 0.882***
Corr: -0.192***
Female: -0.325***
Male: 0.069***
Corr: 0.145***
Female: -0.062***
Male: 0.460***
Corr: -0.351***
Female: -0.496***
Male: -0.155***
Corr: -0.047***
Female: -0.263***
Male: 0.207***
Corr: 0.901***
Female: 0.926***
Male: 0.867***

What affects the net shape of the curve?
A few hypotheses

- Income Inequality
- Educational Inequality
- Ethnic Heterogeneity
- Time Period
- Form of Government
- . . . .
- (watch this space)

Conclusion

- The "Shape" of the mortality curve is a slippery concept, hard to pin down
- Mostly, the shape of the mortality is given by:
  - Biological constraints
  - The overall level of mortality in the population
- Looking for the small differences that are socially meaningful
  - rapid or protracted declines at very young ages;
  - premature or delayed mortality in middle ages;
  - accelerated or reduced ageing in later life

Omnibus Measures

- Describe, in one figure, the distribution and concentration of lives and deaths over the life span
- Some (H_k) are so closely tied to life expectancy that
  - they can offer no new information
  - but may make useful proxies
- Others ($e^\dagger$, H_a)
  - show *some* distribution around the trend line,
  - *may* offer some insights

Parametric models

- Unlikely to be one model to fit all mortality curves.
- Need a serviceable approximation that is
  - easy to fit
  - faithfully reflects the general properties of the curve.
- The hexic curve

$$Y = X^6 - \beta X^4 + \emptyset X^2 + \tau X$$

suitably scaled and located,

- provides a serviceable approximation
- with interpretable parameters

WARNING

- Having a lot of good data, a sound theoretical foundation and a credible model
  - Is no guarantee that we're going to find a meaningful answer
    - **Caveat User**

Thank You
Gracias
Eskerrik Asko

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