# Further applications of the multistate life table decomposition method

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# Some changes to our focus

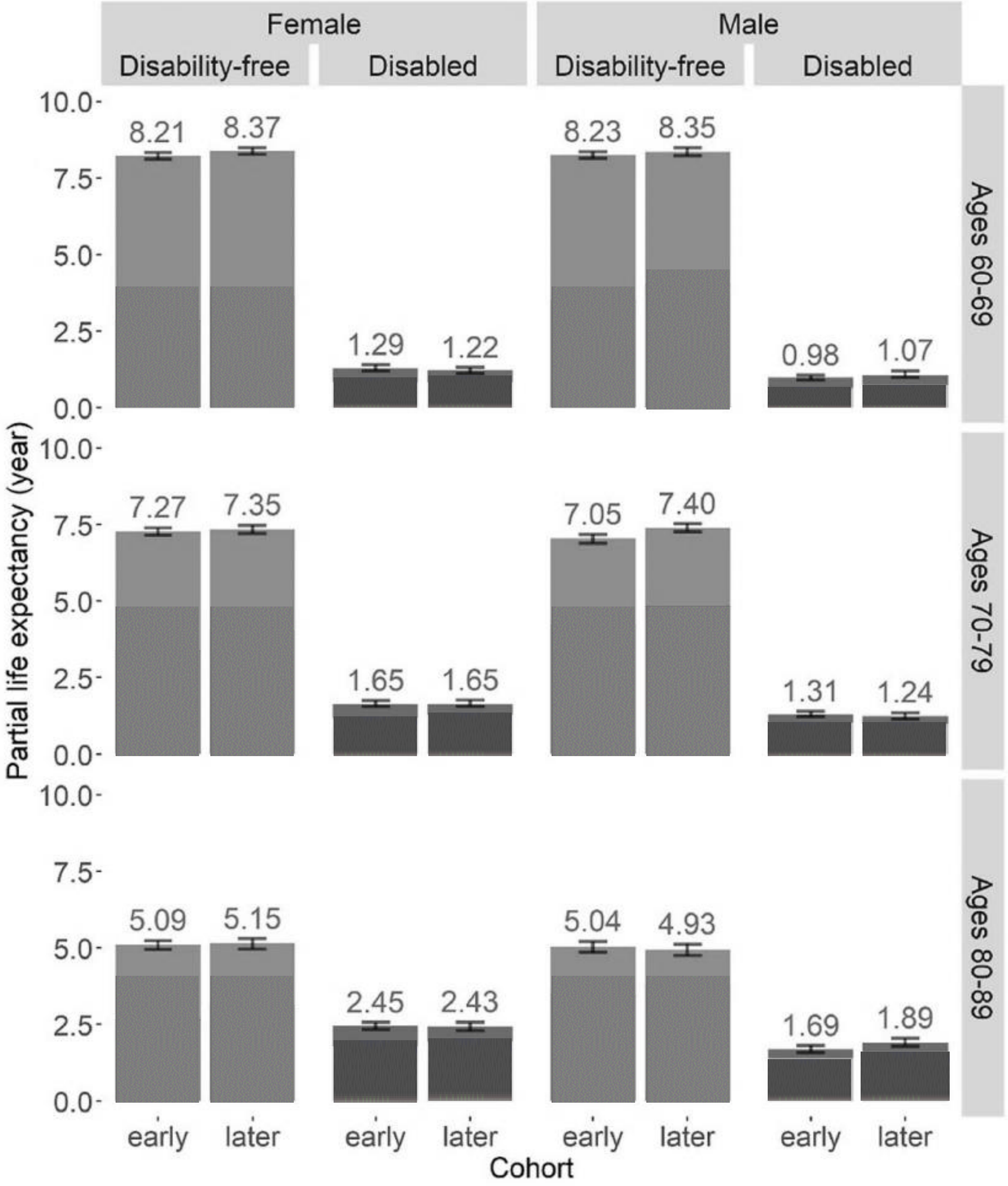
*Decoupling Disease and Disability: A cohort analysis of healthy longevity in the US and UK*

- No mortality in ELSA after 2012? No cross-national view so far…
- A failed attempt to decompose things…

- Interesting issue emerges when we explore this question
- Now it is more of a method-focused paper

# Background

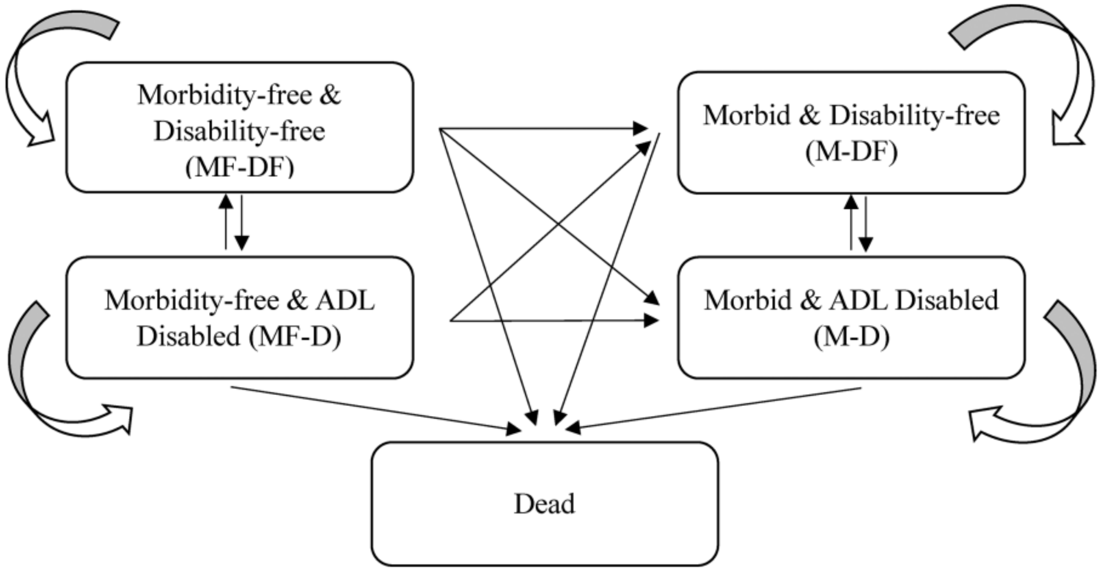
- Here is the change in 10-year cohort life expectancy of two consecutive US birth cohorts of different age ranges
- Disability-free Life expectancy (DFLE) has small increase typically, also not quite significant
- Yet, Disabled Life Expectancy (DLE) doesn't necessarily decrease. For male in some ages, the DLE even increase slightly




Source: Shen & Payne (2023)

# Background

- Using a five-state multistate model, this paper looks at the interaction between morbidity and disability

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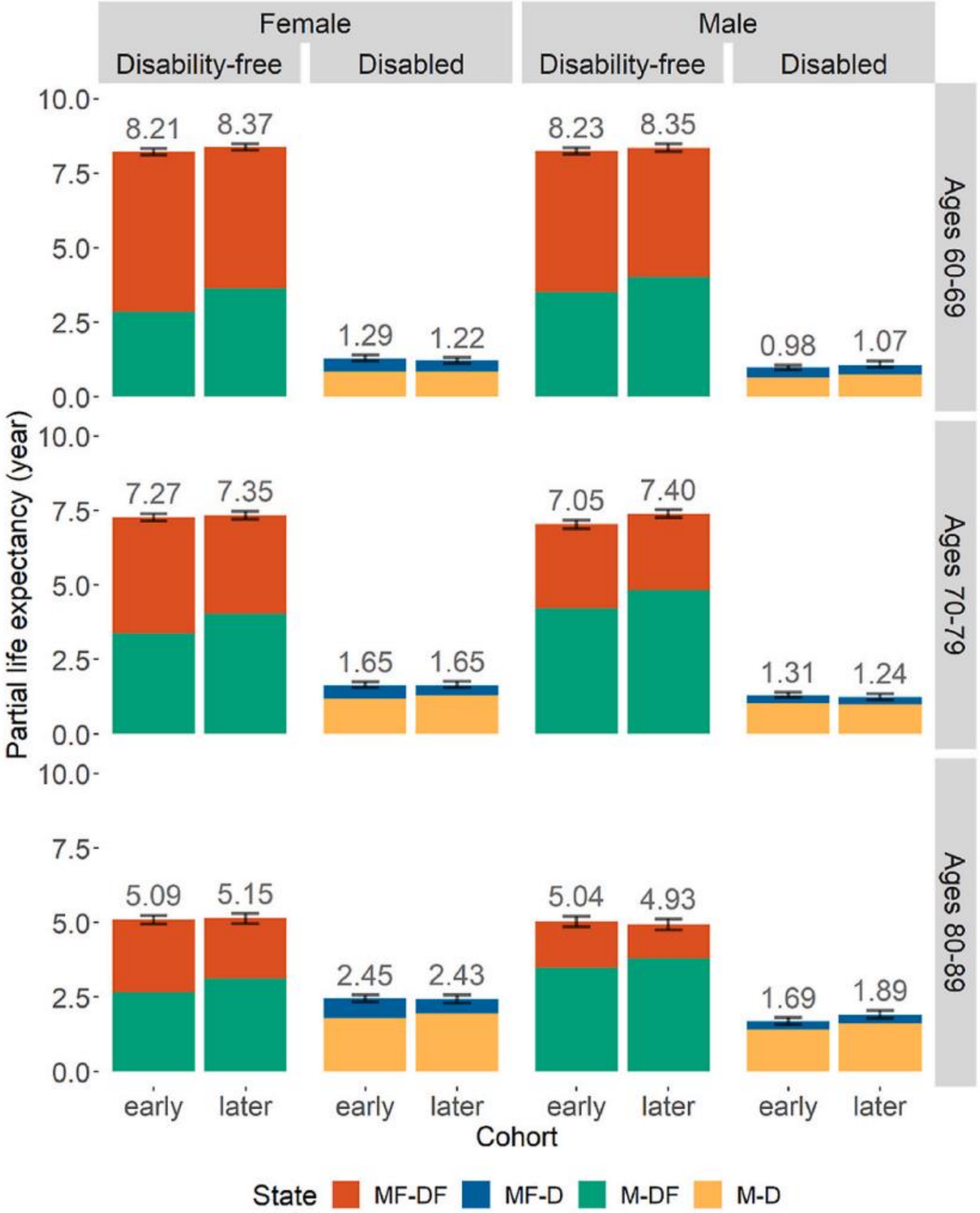
# Background

MF - Morbidity free;
DF - Disability free;

M - Morbid
D – Disabled

- Using a five-state multistate model, this paper looks at the interaction between morbidity and disability
- Within DFLE, average individual are spending significantly more time with morbidity (in green) over cohort
- Within DLE, the time (in yellow) with morbidity didn't change too much
- This indicates a dynamic equilibrium

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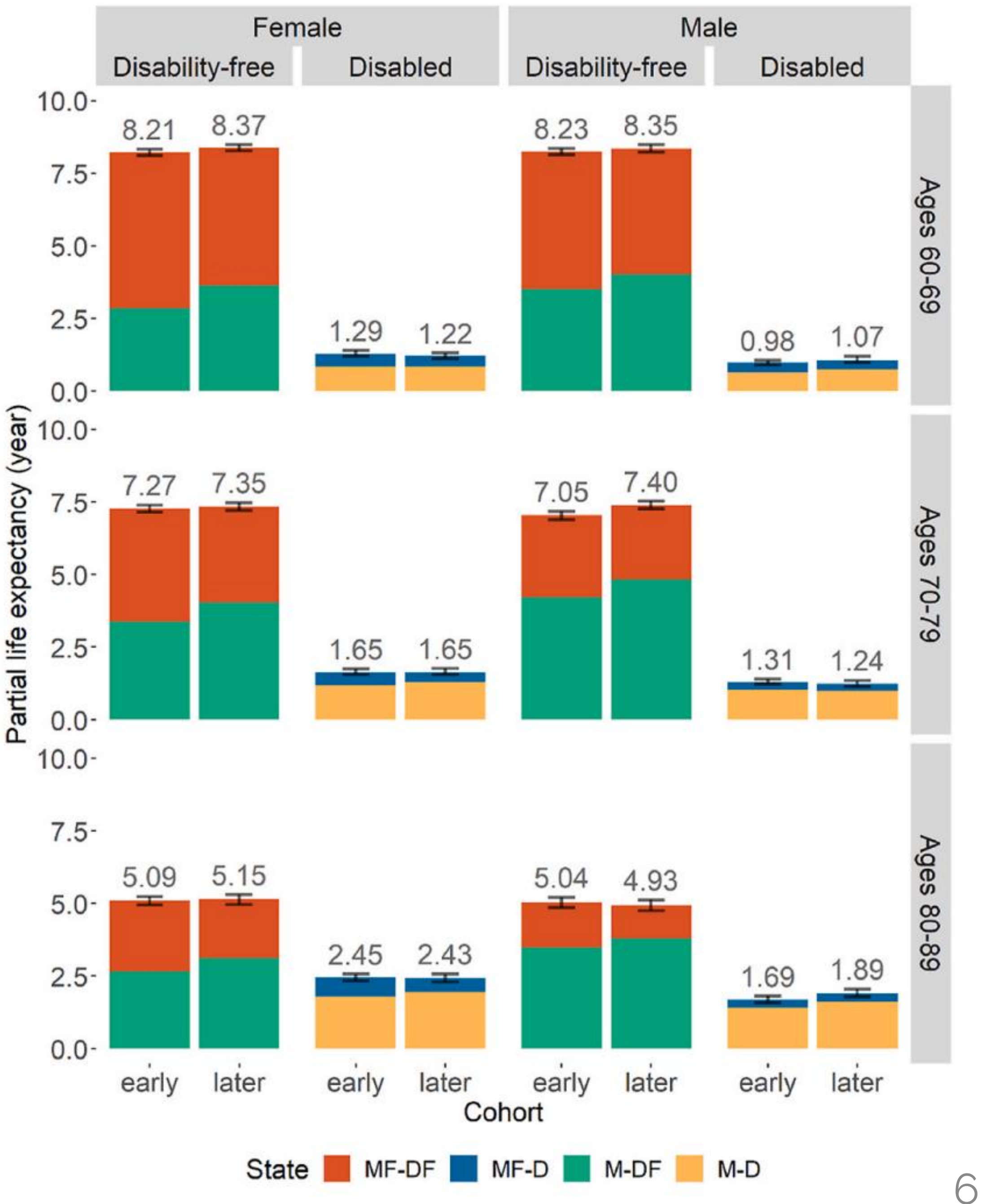
# The research question

*What is the effect of the changing morbidity on the stagnation in Disability-free (and Disabled) Life Expectancy?*

- We have a multistate life table decomposition method (Shen et al. 2023), but if we decompose this five-state multistate, we cannot distinguish the effect from morbidity
- If morbidity status doesn't change within the age range, we can treat them as two groups of people with the weight of the prevalence. For example, with the education groups in another paper (Shen et al. 2025)
- Therefore, we need a new method

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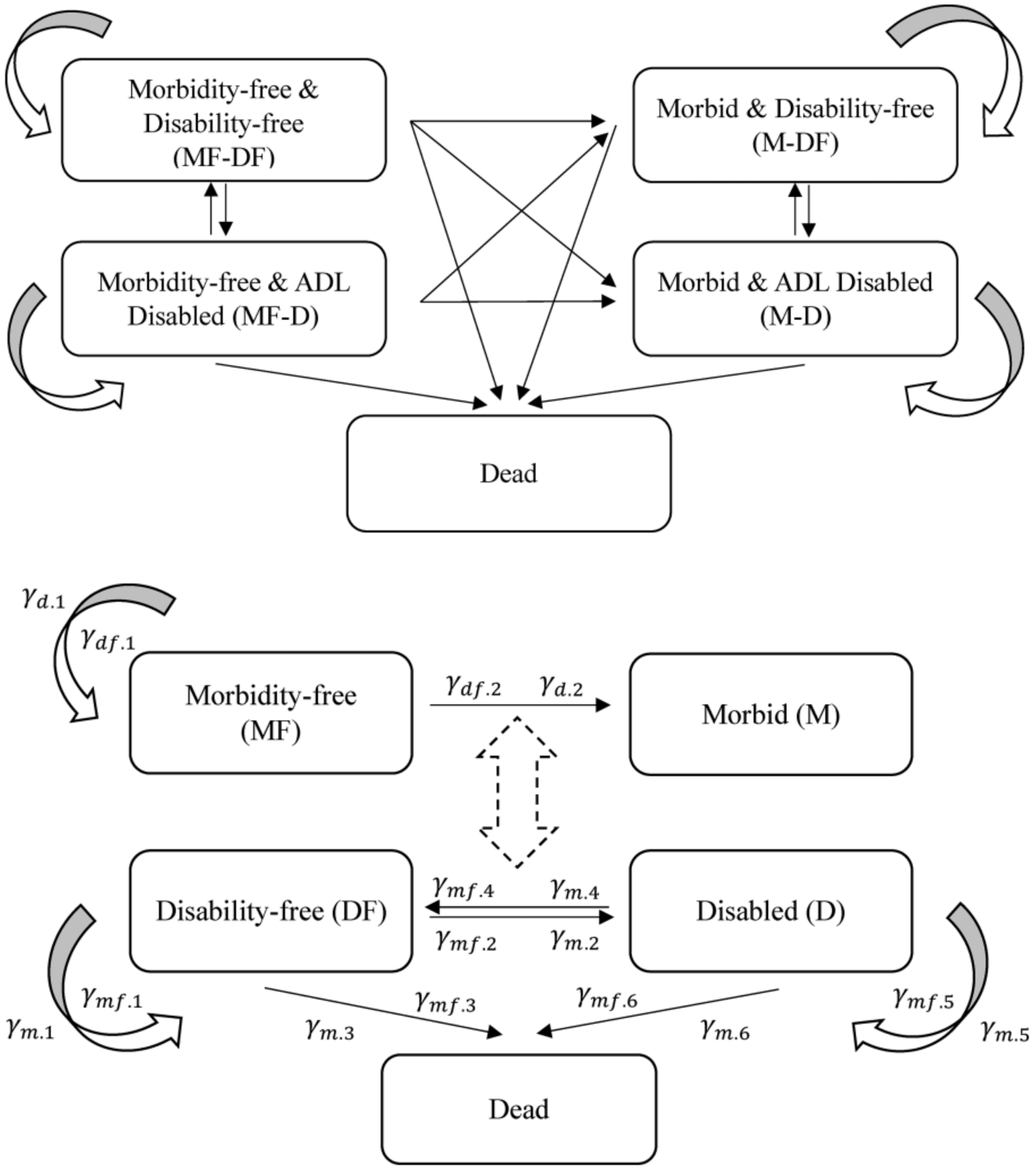
# Method

## Multiple Multistate Method (MMM):

- This method provides a flexible yet simple way to model two or more time-varying variables
- The transition of the morbidity and disability are separately modelled conditioned on the current state of both domains
- The two models are combined by a simulation
- We can decompose this system into
  1. Effect from morbidity
  2. Effect from disability

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# Method

## Decomposition of the gap in two life expectancies

(Shen et al. 2023)

$$ {}_{\beta-\alpha}\dot{\mathbf{e}}_{\alpha} = \frac{\dot{\mathbf{i}}_{\alpha}}{2} + \sum_{x=\alpha+1}^{\beta-1} \dot{\mathbf{i}}_{x} + \frac{\dot{\mathbf{i}}_{\beta}}{2} $$

$$ = \underbrace{\dot{\mathbf{i}}_{\alpha}\; {}_{\beta-\alpha}\mathbb{e}_{\alpha}}_{\text{Baseline population structure}} + \underbrace{\sum_{x=\alpha}^{\beta-1} \mathbf{l}_{x}\dot{\mathbf{P}}_{x} \cdot \left(\frac{\mathbb{I}}{2} + {}_{\beta-x-1}\mathbb{e}_{x+1}\right)}_{\text{The transition matrices at each age}} $$

*Baseline population structure* *The transition matrices at each age*

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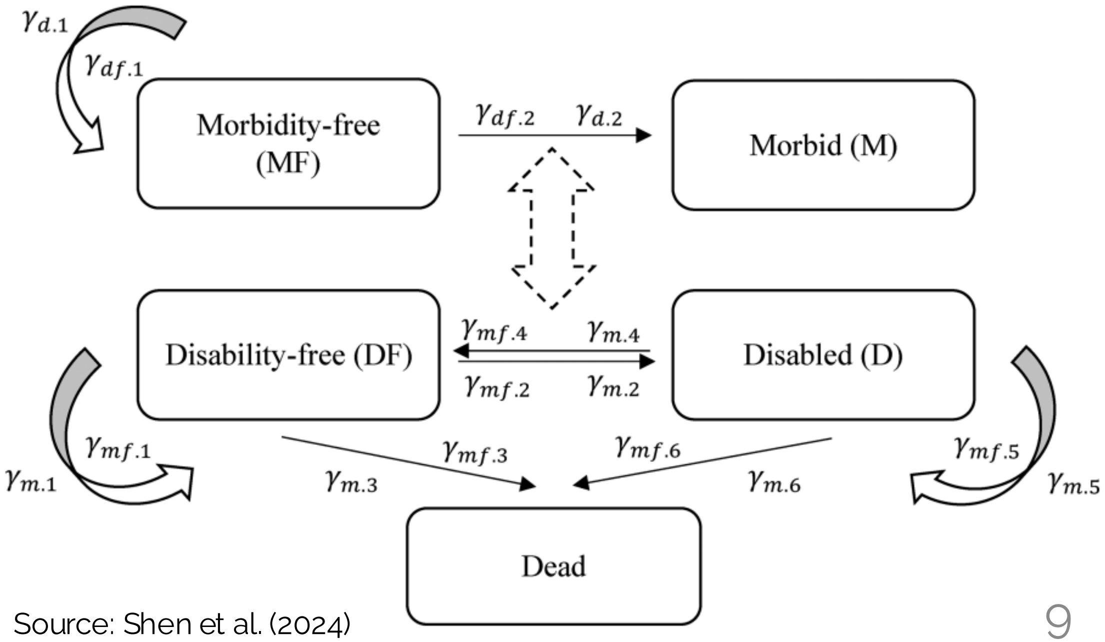
*Baseline population structure*

*The transition matrices at each age*

## Multiple Multistate Method (MMM)

- We can decompose this into this system into
  1. Baseline from morbidity
  2. Baseline from disability
  3. Transition matrix from morbidity
  4. Transition matrix from disability
     - Conditioned on morbidity-free
     - Conditioned on morbid

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# First Attempt

## Generalized decomposition methods with simulation

- Stepwise Decomposition Method
  - Switching the variables one by one in vector A -> vector B, and compute the life expectancy each time
- Horiuchi's Method
  - Switching the variable one by one from A -> B by small interval (1%), and compute the life expectancy each time
- Both methods require computing the life expectancy many times
  - More in Horiuchi
- The order of which variable in the vector is changed first would affect the decomposition outcome, and increase the number of computation

*Quite time-consuming even without a simulation*

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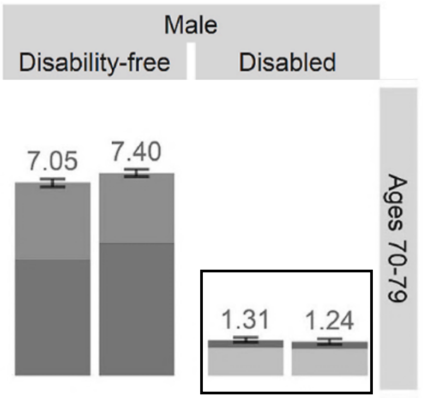


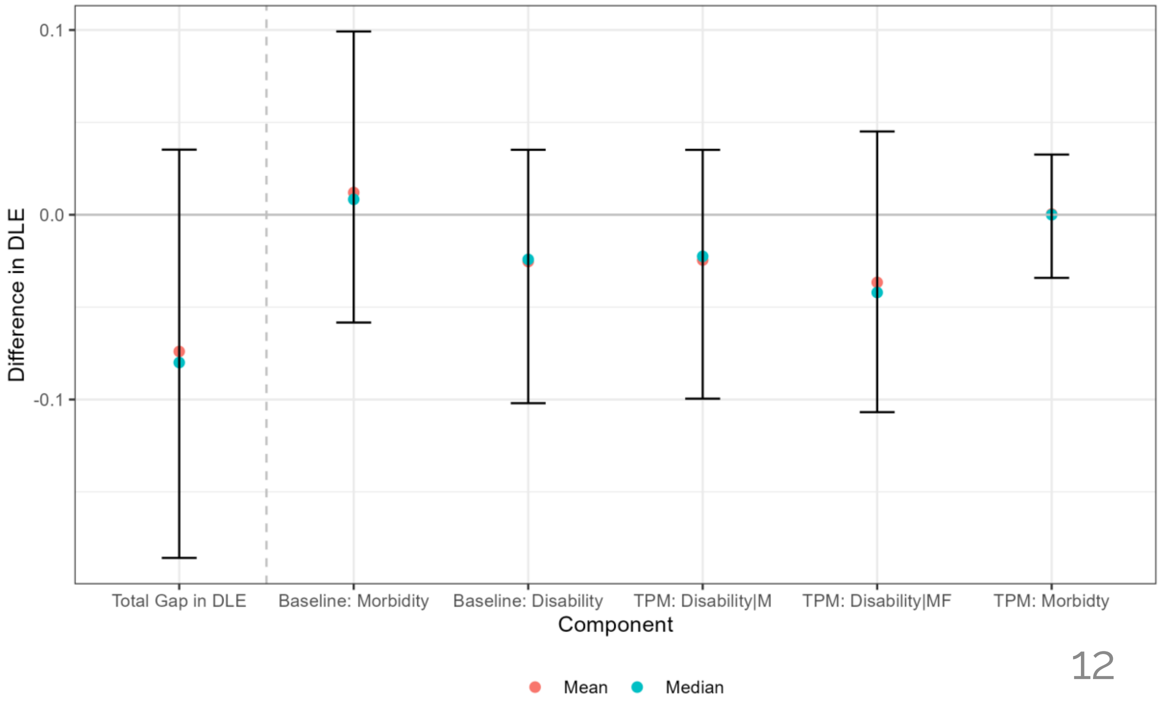

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2. Stochasticity in the simulation (producing different results each time with the exact same inputs)
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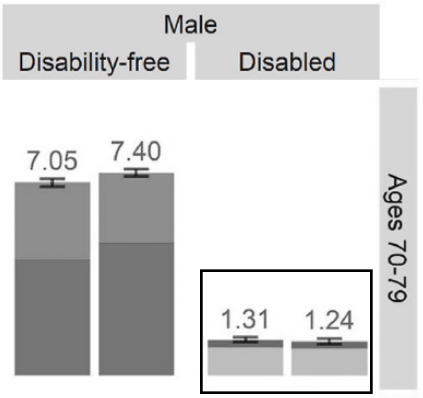
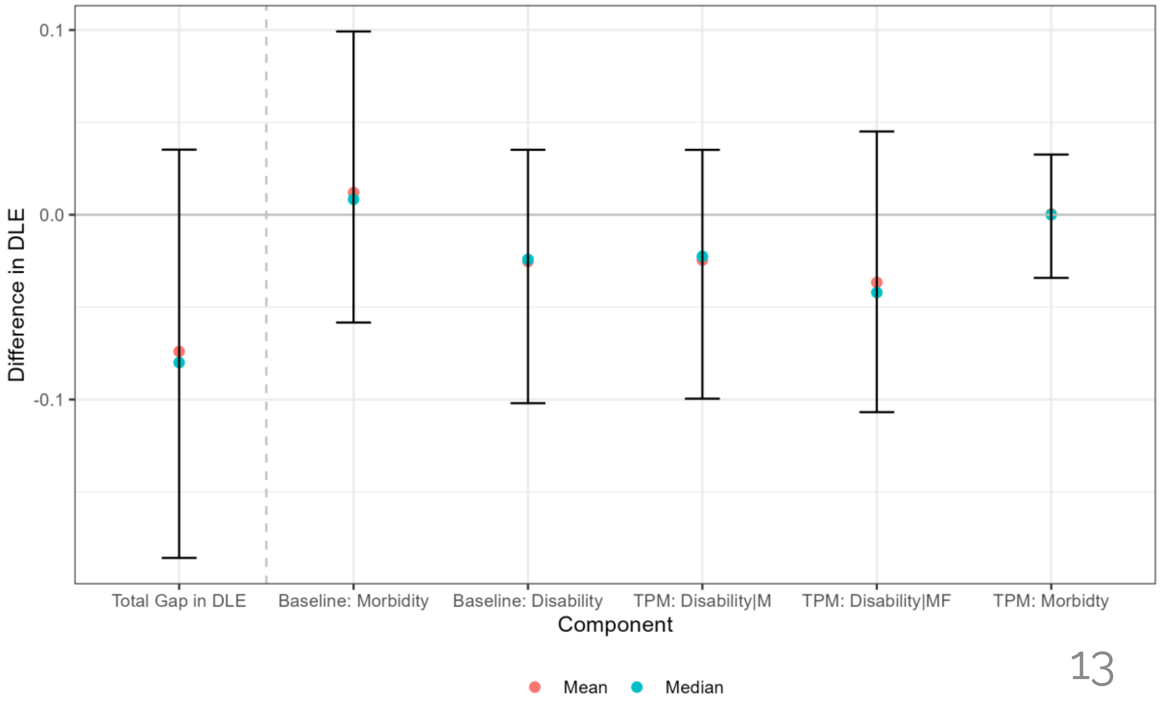
   - The total probability after swapping the values between two variables might be above 1
   - Horiuchi will take forever because it has more incremental steps



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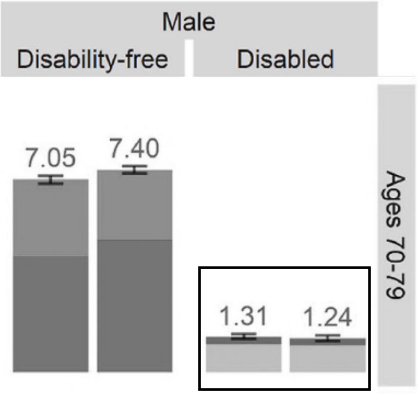
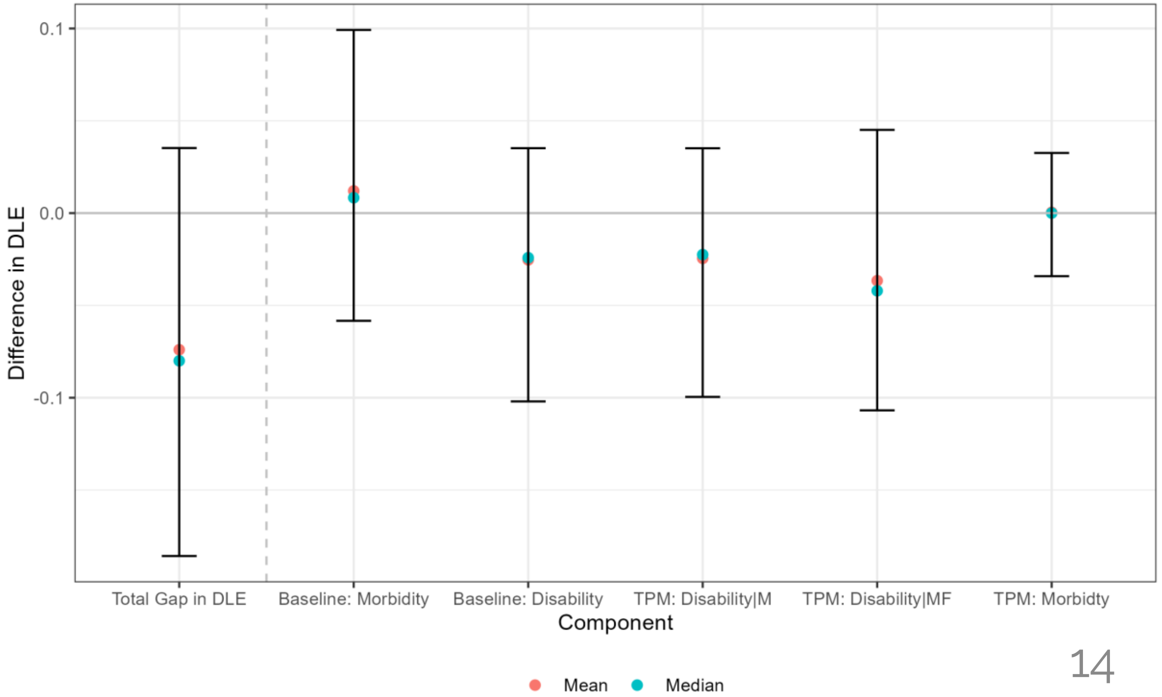
Generalized decomposition methods with simulation is **not a great idea** (or at least inefficient)

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# Re-attempt: Analytical solution

Decomposition of the gap in two life expectancies
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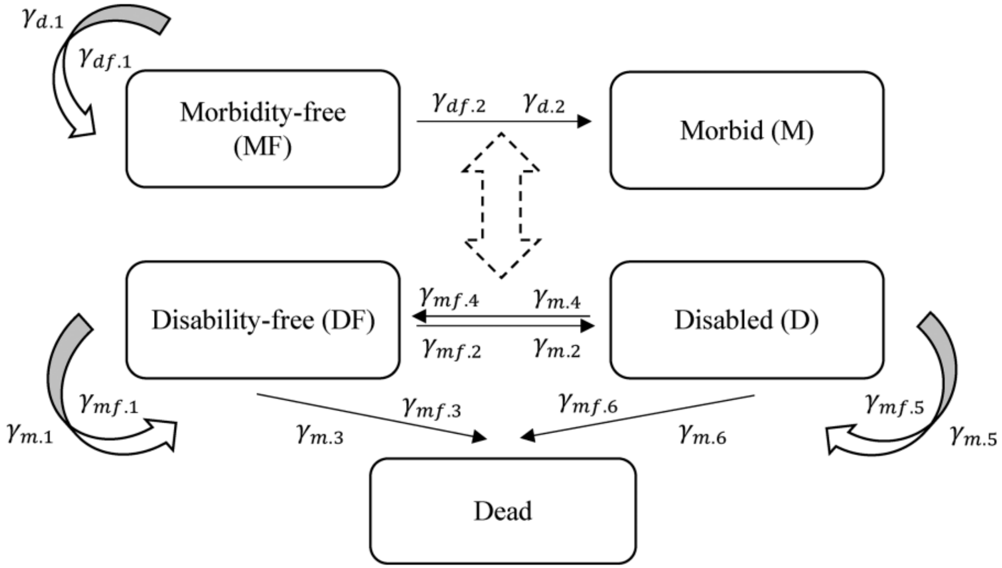
*Baseline population structure* *The transition matrices at each age*

Multiple Multistate Method (MMM)

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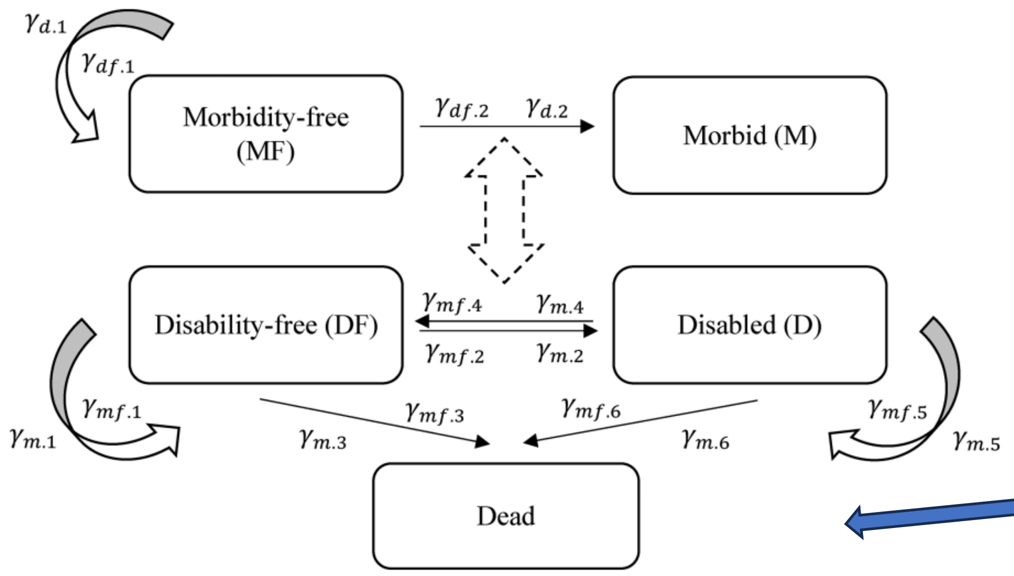


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In MMM, the transition to dead can be in either system.

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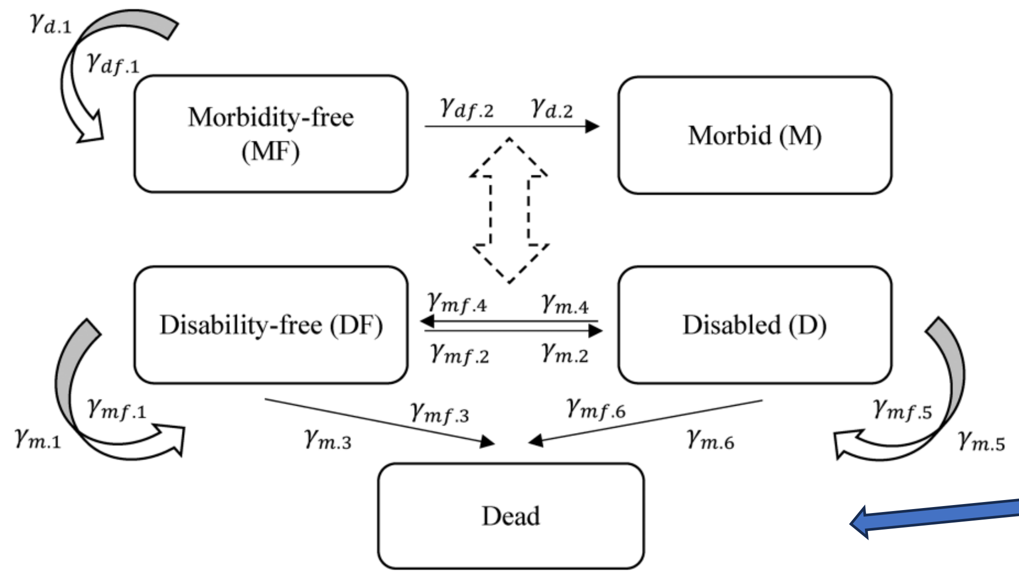


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$$ \mathbf{P}_{x} = \mathbf{P}_{x}^{morbdity} \circ \mathbf{P}_{x}^{disability} \circ \mathbf{P}_{x}^{mortality} $$

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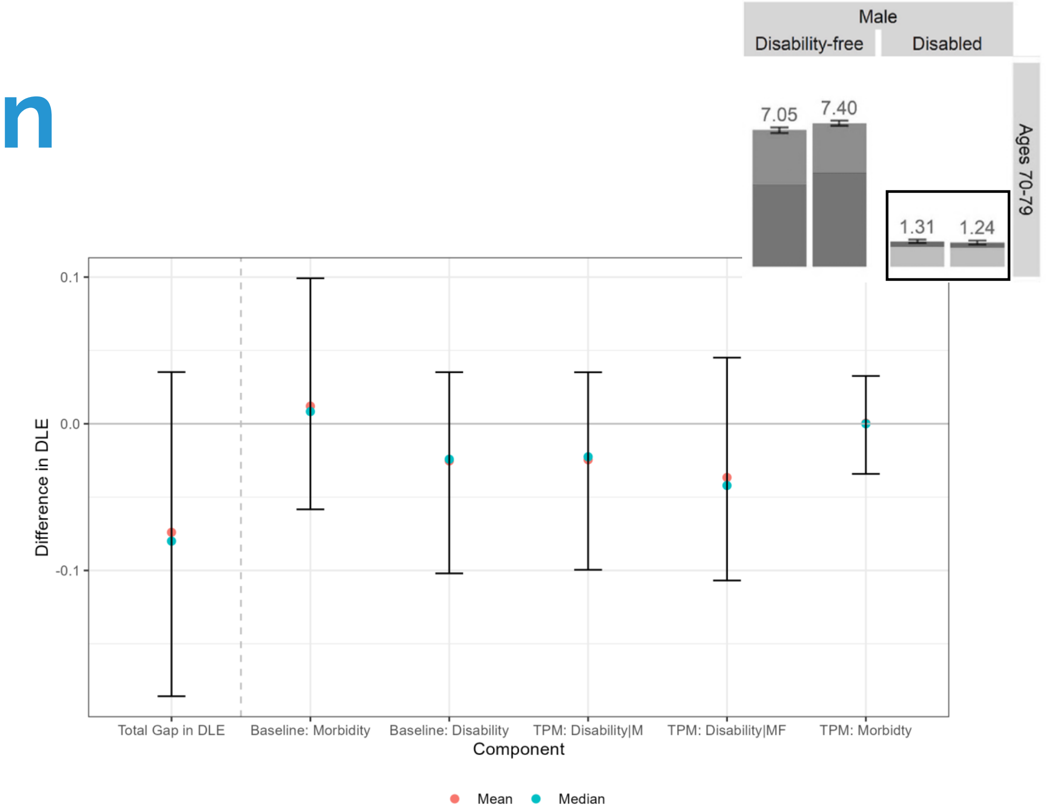
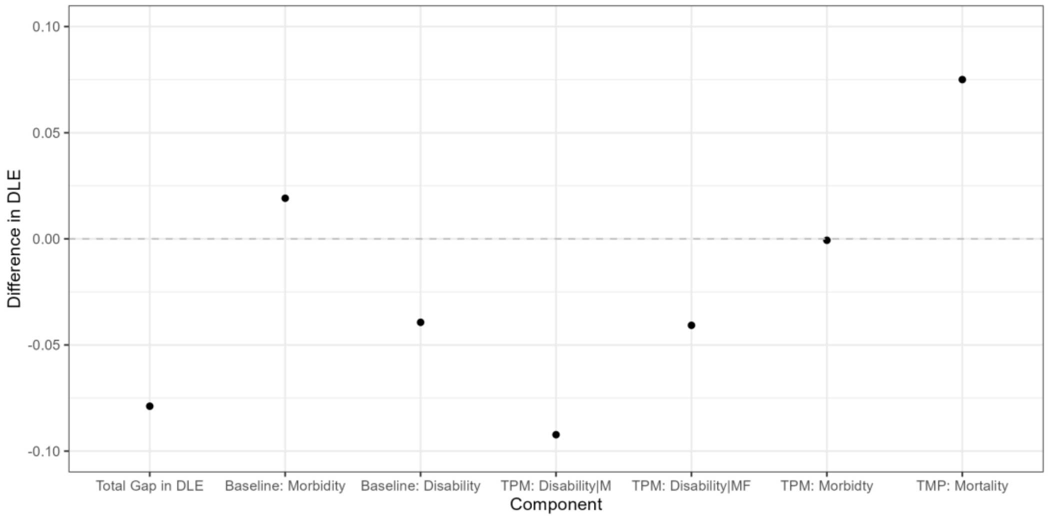
- Compare the results from General method and analytical solution
- Both results are produced by the same input of baseline and transition probability
- Values are very similar, and there is no variability from the analytical solution

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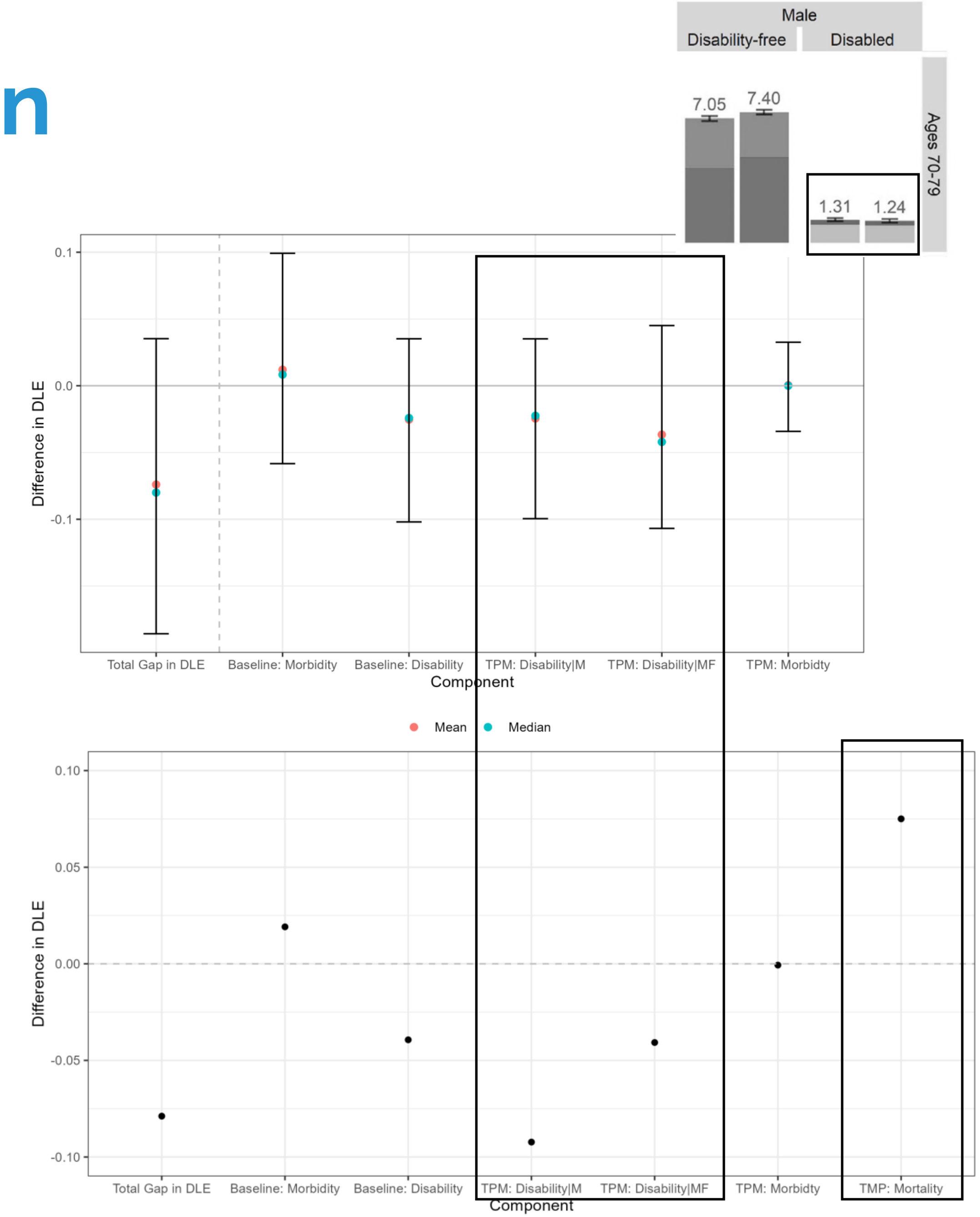
- The survival advantage in later cohort would lead to higher DLE



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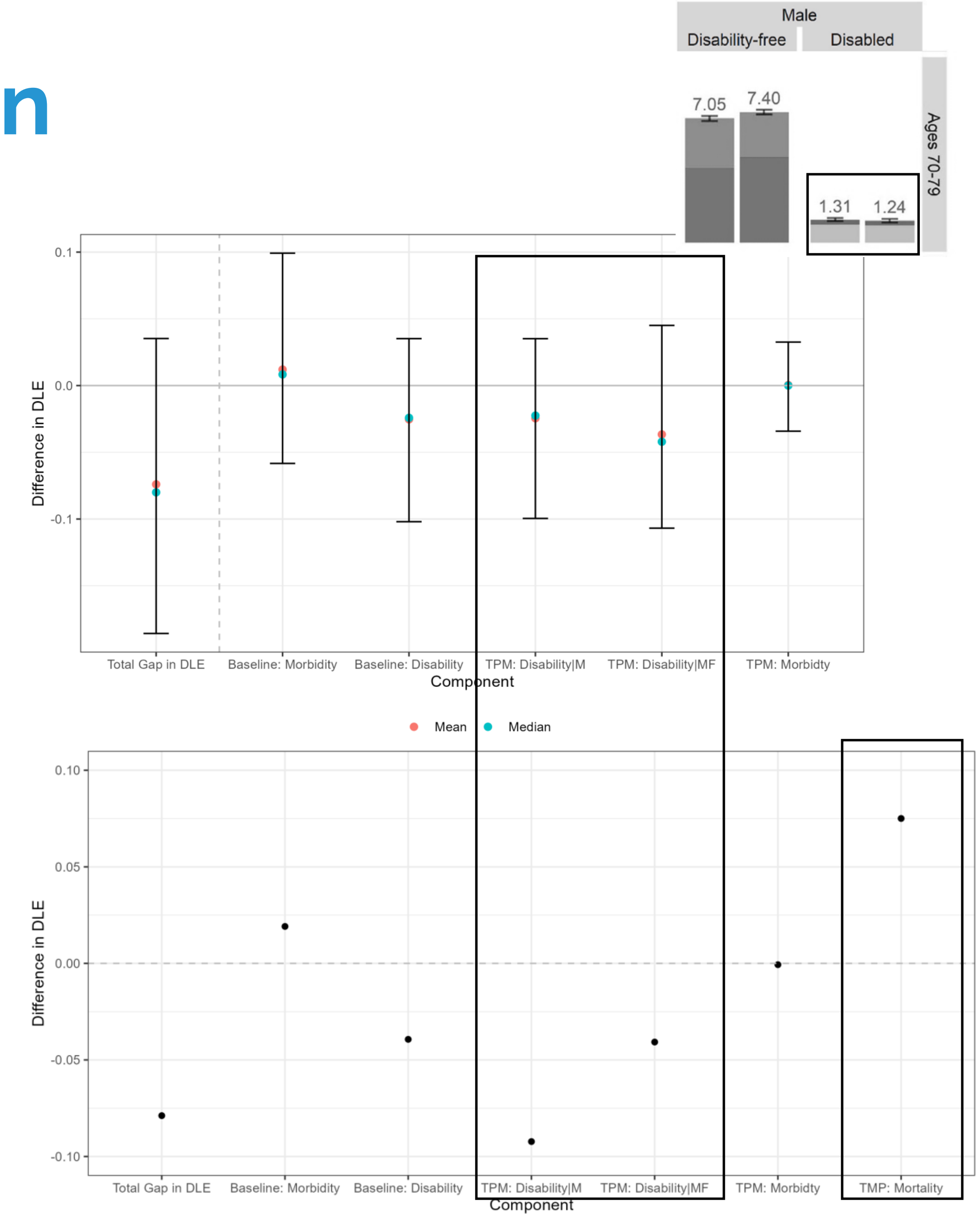
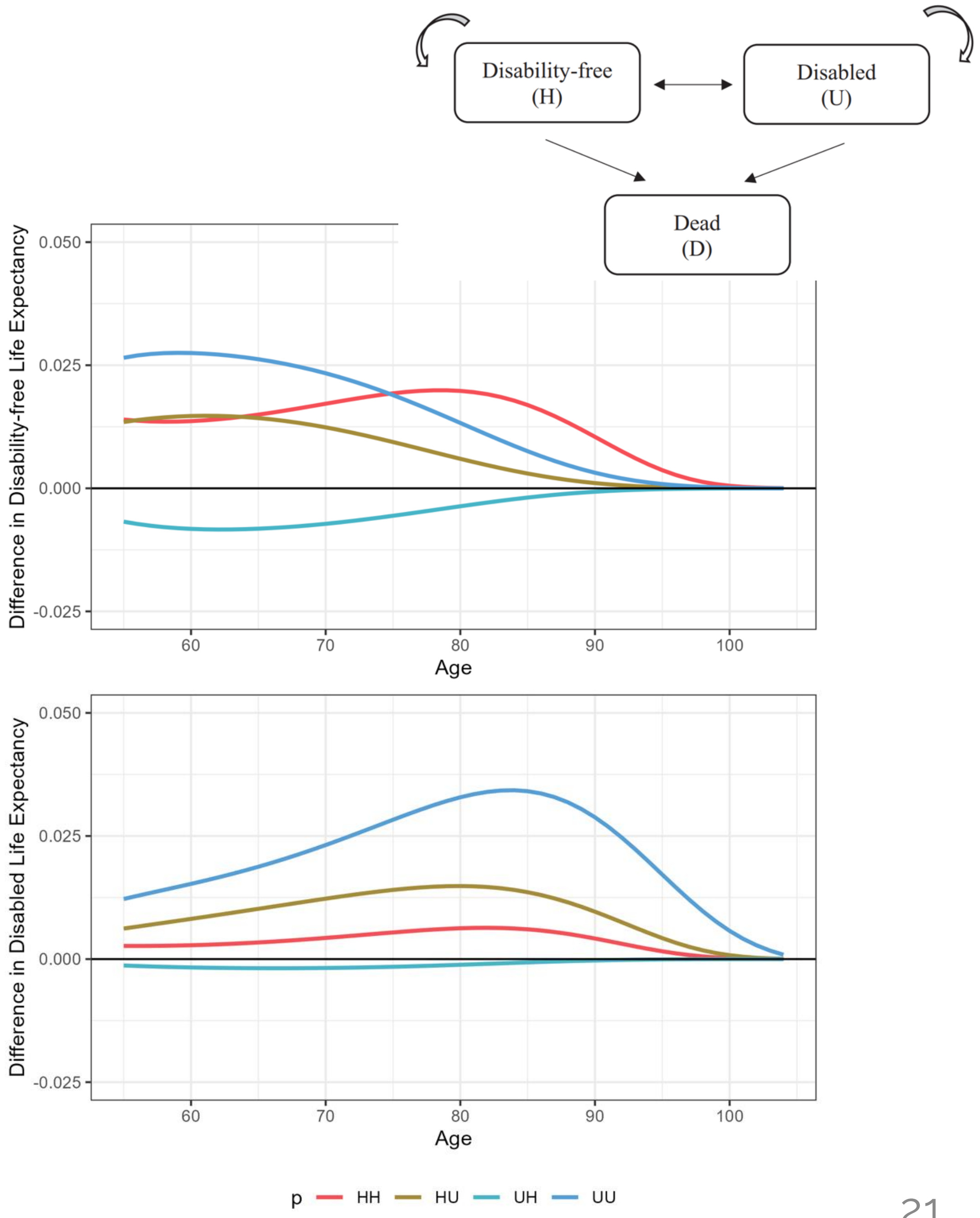
This inspires an interesting thoughts for the interpretation of the original multistate life decomposition method



# Further remarks

- Here are the results from the original multistate decomposition method
- It is a typical three states model
- It is comparing the healthy life expectancy gap between women and men

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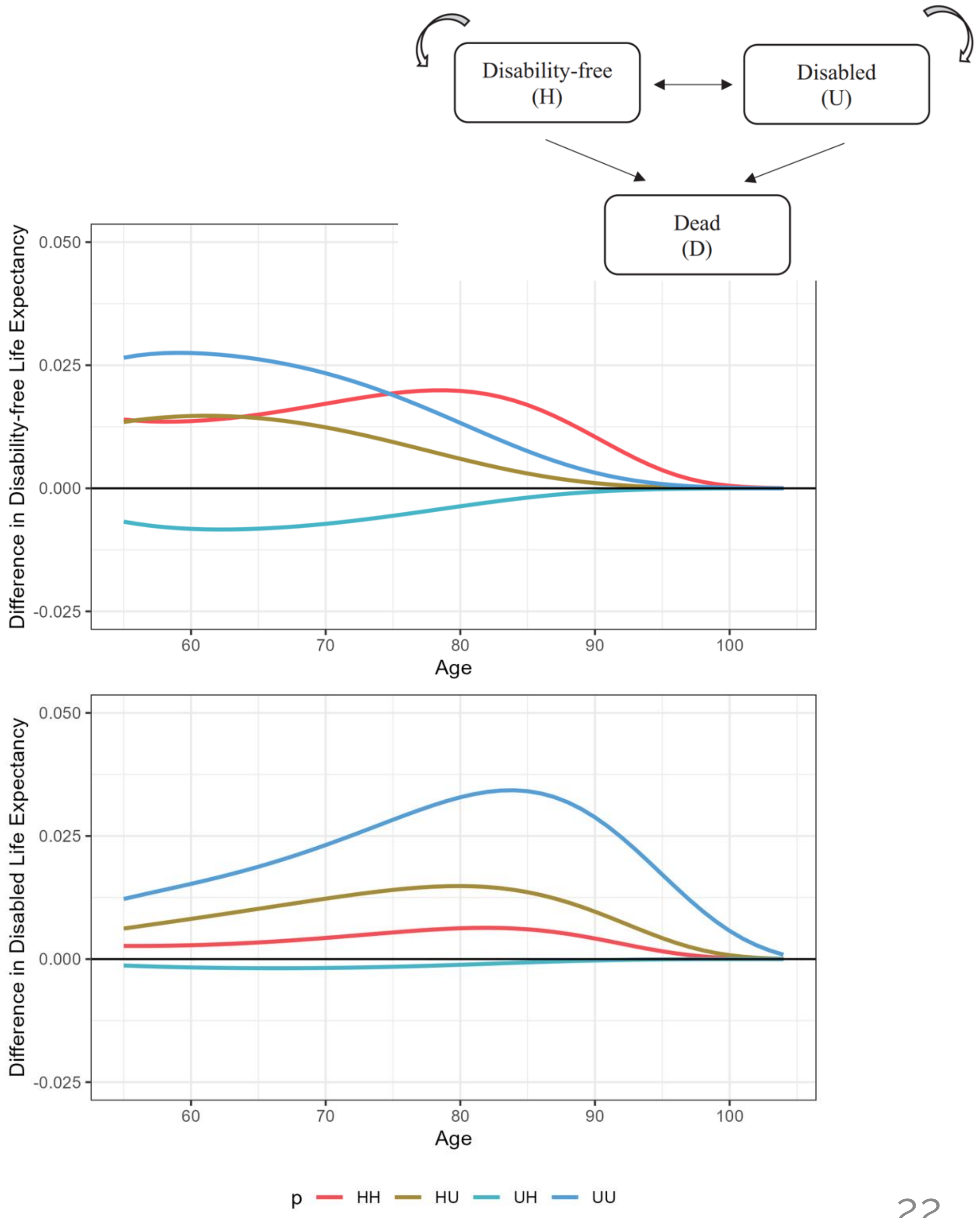
- There are 6 transition probabilities but, in the figure, we only see 4 of them (HH, HU, UH ,UU)

*What about the effect of mortality (HD and UD)?*

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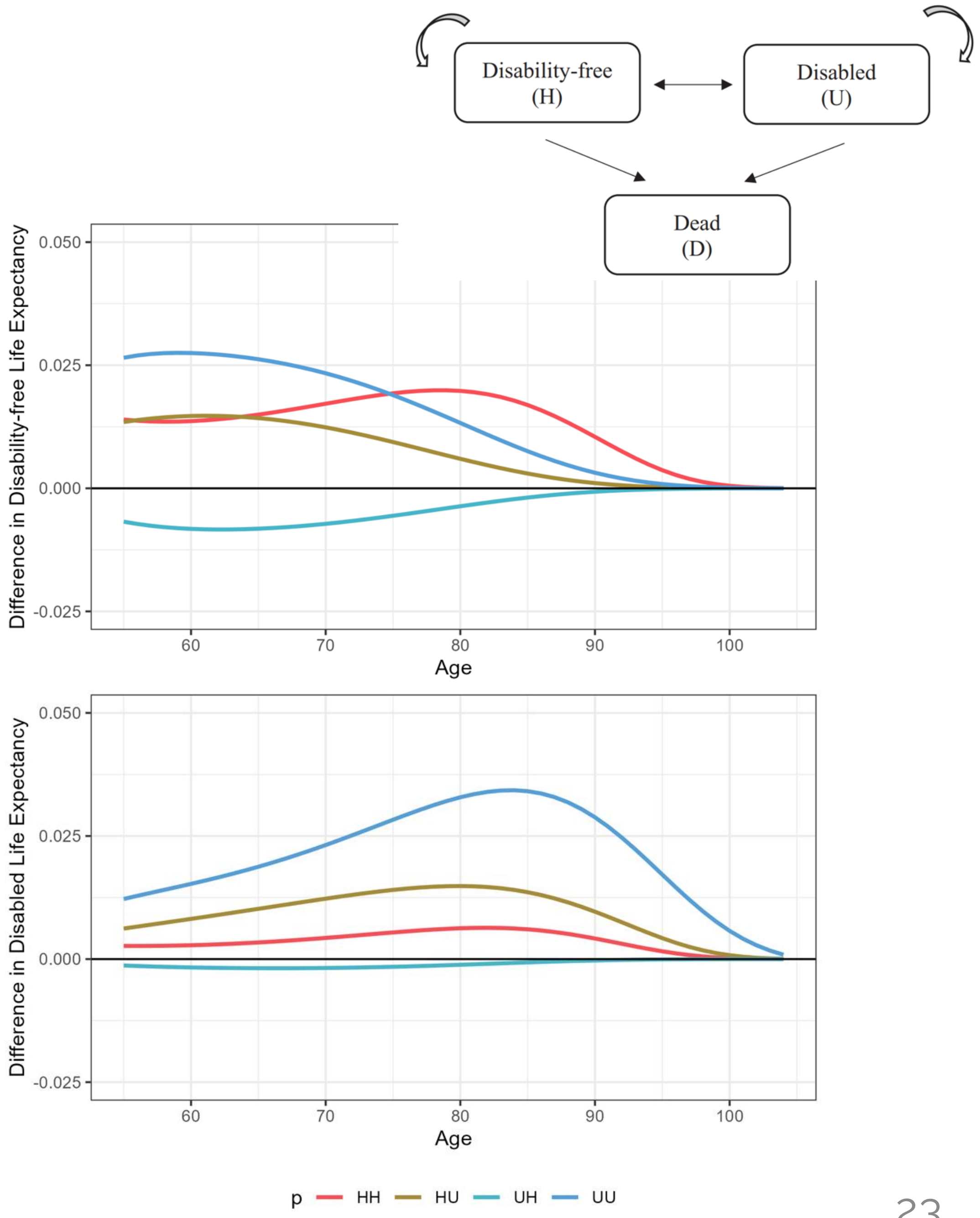
*What about the effect of mortality (HD and UD)?*

- The effect from HD is embedded in HH and HU
  - Positive HH and HU for women would indicate that women has higher survival
- However, it is not quite clean and satisfying…

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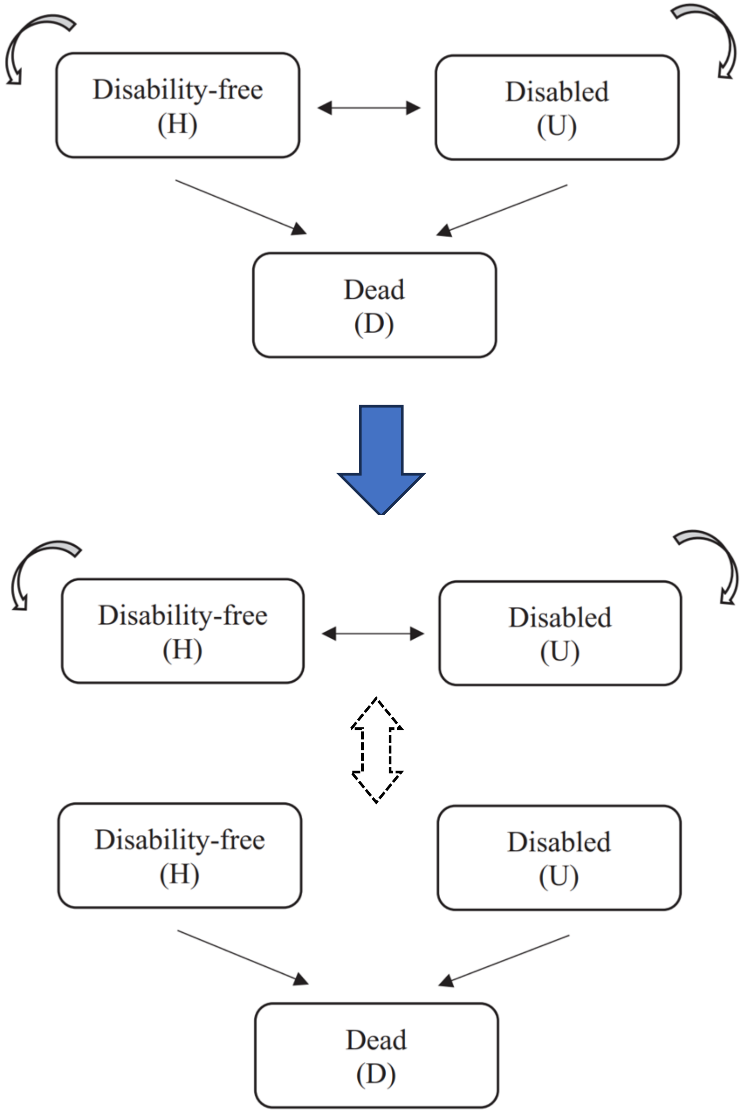
- Then, we can decompose the effect from each system



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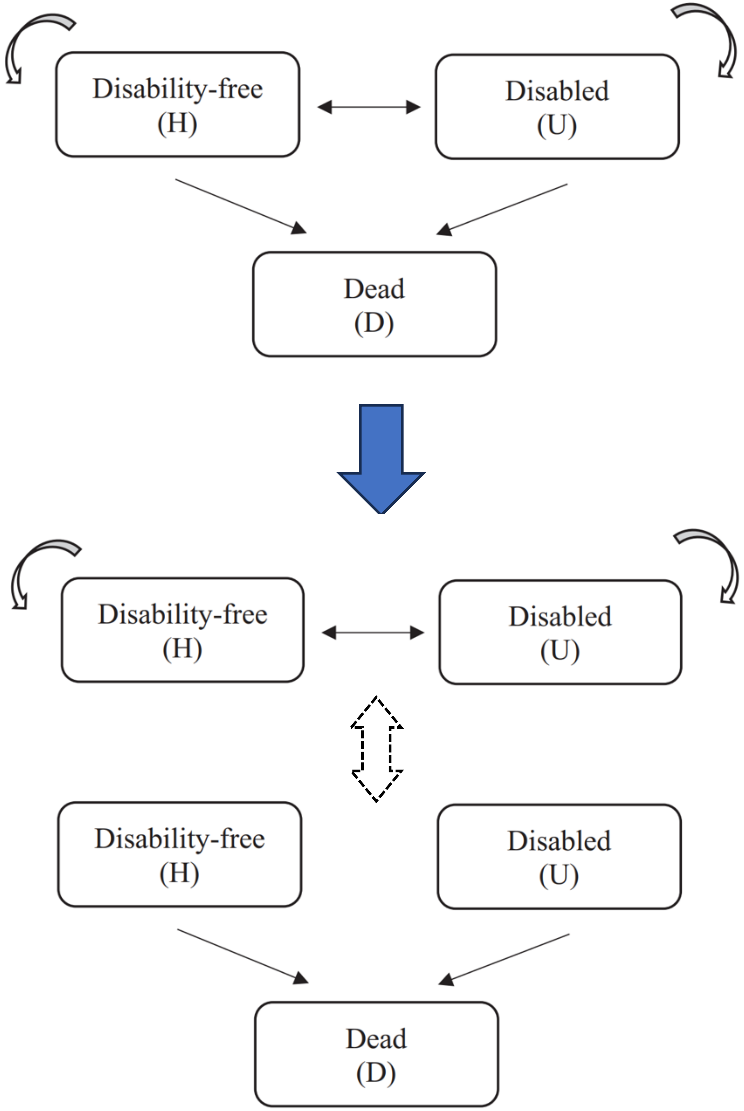
- The interpretation is: conditioned on survival, what are the effects from the transition probability between states on the difference in life expectancy
- The concept and results is more comparable to the decomposition from Sullivan's method



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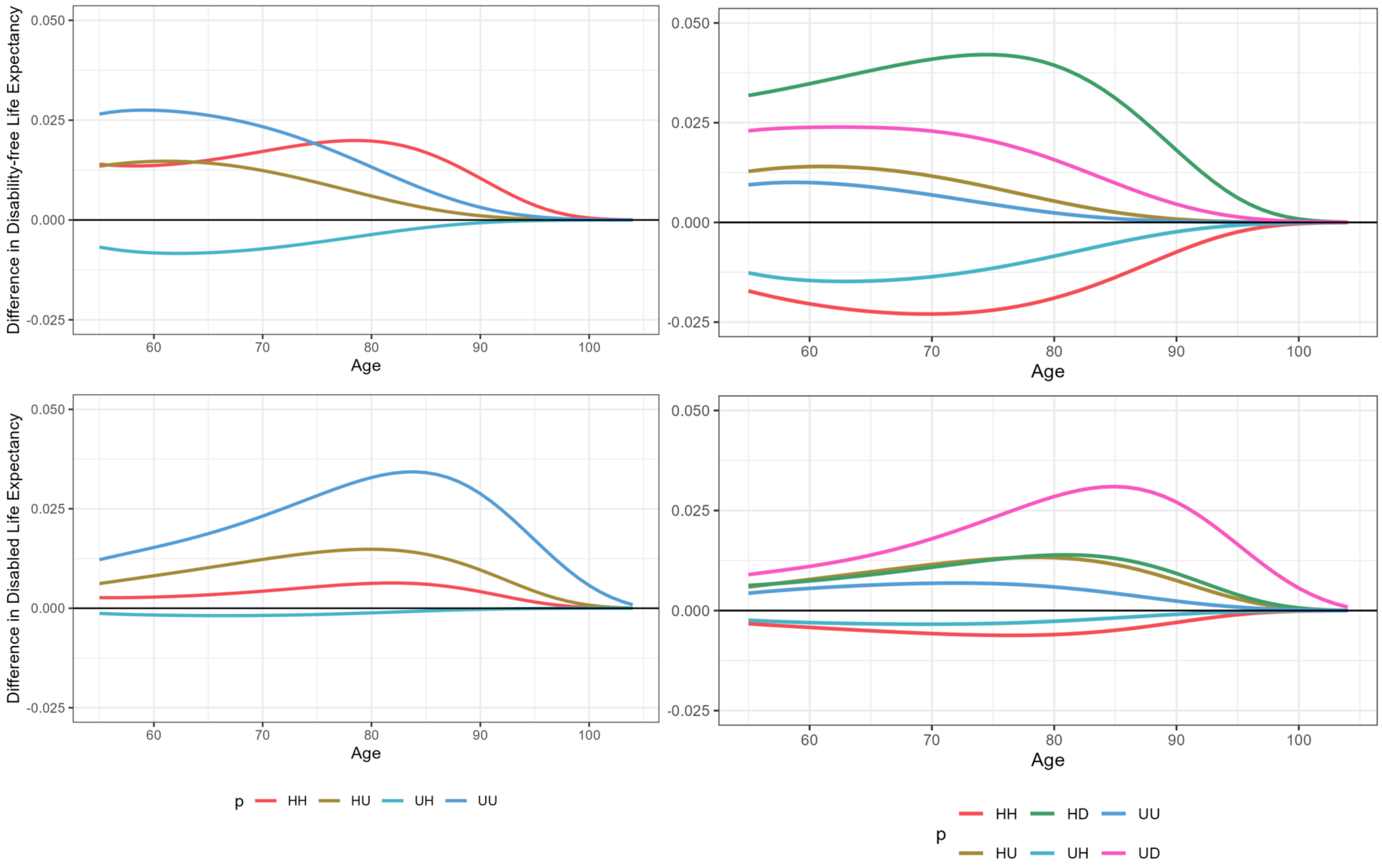
Original | MMM



# Conclusion

- Further develop the multistate life table decomposition method

1. Widen the use case with MMM and can be used to understand other population composition effect that changes over time
2. Improve the interpretability of the results after decomposing the mortality effect

# Further applications of the multistate life table decomposition method

Thank you and your comments are very welcomed!